# NS Ghost Slivers

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**Abstract**

Neveu-Schwarz ghost slivers in pictures zero and minus one are constructed. In particular, using algebraic methods $\beta$, $\gamma$ ghost sliver in the $-1$ picture is obtained. The algebraic method consists in solving a projector equation in an algebra, where the multiplication is defined by a pure 3-string vertex without any insertions at the string midpoint. We show that this projector is a sliver in a twisted version of $\beta$, $\gamma$ conformal theory. We also show that the product of the twisted $b$, $c$ and $\beta$, $\gamma$ ghost slivers solves an equation that appears after a special rescaling of super VSFT.

# Contents



# 1 Introduction.

During the last year the bosonic vacuum string field theory (VSFT) proposed to describe physics around the bosonic tachyon vacuum [1] has been investigated in many papers [2]-[45]. By product this study has revealed many interesting features of string field algebra. The characteristic feature of VSFT is a very simple form of the kinetic operator $\mathcal{Q}$. This $\mathcal{Q}$ is just $c(i)-c(-i)$ in the bosonic case. It is apparently related by a singular transformation with the shifted BRST operator, $Q = Q_B - [A_0, \cdot]$, where $A_0$ is a vacuum solution of the open string field theory (SFT) describing the D-brane decay. Due to an absence of a dependence of $\mathcal{Q}$ on the matter fields VSFT equations of motion admit a factorized form with the projector-like matter part. Solutions of projector equations have been discussed in many details [2]-[41]. These equations are similar to the non-commutative soliton equations in the large non-commutativity limit [50].

Ghost part of VSFT equations of motion has been studied in [15, 16, 19, 21, 3, 27, 32]. It was observed that a sliver constructed in the twisted conformal theory with new $SL(2,\mathbb{R})$ invariant vacuum solves the ghost part of VSFT equation of motion. This equation is a usual SFT equation of motion with a canonical choice of ghost kinetic term that is a local insertion at the string midpoint.

It is interesting to consider a generalization of these treatments to the case of superstrings. A generalization of VSFT to superstrings has been discussed in [1] and more recently in [39] and [40] in the context of cubic SSFT [53, 54] and non-polynomial SSFT [51], respectively. Fermionic projectors, such as the NS sliver, have been constructed in [39, 40]. Although in the matter sector we have standard equations for projectors, in the ghost sector we have to solve a bit more complicated equation. As it has been noted in [41], the VSFT kinetic operator $\hat{\mathcal{Q}}$ in superstring case inevitably has the matrix structure, or in component notations has a non-diagonal form and mixes GSO$\pm$ sectors:

$$\hat{\mathcal{Q}} = \begin{pmatrix} \mathcal{Q}_{\mathsf{odd}} & \mathcal{Q}_{\mathsf{even}} \\ -\mathcal{Q}_{\mathsf{even}} & -\mathcal{Q}_{\mathsf{odd}} \end{pmatrix}, \tag{1.1}$$

with $\mathcal{Q}_{\mathsf{even}} \neq 0$. This is related with the fact that the corresponding $A_0$ describing a decay of a non-BPS brane has non-zero GSO$-$ component. If $\mathcal{Q}_{\mathsf{even}}$ were zero, we could take $\mathcal{Q}_{\mathsf{odd}}$ to be the ghost kinetic operator used in the bosonic VSFT [3]. In [41] the following candidates for $\mathcal{Q}_{\mathsf{odd}}$ and $\mathcal{Q}_{\mathsf{even}}$ have been proposed:

$$\mathcal{Q}_{\mathsf{odd}} = c(i) + \frac{1}{2\pi i}\oint b(z)\gamma^2(z)dz, \qquad \mathcal{Q}_{\mathsf{even}} = \gamma(i). \tag{1.2}$$

Therefore, VSFT equations of motion in supersting case are more complicated in comparison with their bosonic analog. Under an assumption of a special splitting of these equations one however can get projector-like equations with a ghost insertion. To solve these equations it is reasonable to consider slivers in different pictures. A presence of different pictures (Bose seas) is a special feature of the $\beta$, $\gamma$ system.

A study of superghost slivers and wedge states is a subject of the present paper. We perform this study using generalizations of different technics known in bosonic case. Sometimes these generalizations are straightforward, but sometimes they need new calculations.Since the VSFT in the ghost sector deals with the projector-like equation with an insertion it is appropriate to study slivers in different pictures.

We start from an algebraic method. The algebraic method consists in solving a projector equation in a $*'$-algebra,

$$|\Xi^{\beta\gamma}\rangle *' |\Xi^{\beta\gamma}\rangle = |\Xi^{\beta\gamma}\rangle. \tag{1.3}$$

where the multiplication $*'$ is defined by a pure 3-string vertex without any insertion, $*$ is reserved for a standard multiplication with a suitable ghost insertion [46, 53, 54, 49]. Then we construct a twisted CFT for $\beta$, $\gamma$ ghosts ("CFT in $-1$ picture") and show that twisted $\beta$, $\gamma$ sliver solves the projector-like equation (1.3).

We will argue that the direct product of twisted $b, c$ and twisted $\beta$, $\gamma$ slivers

$$|\Xi\rangle = |\Xi'_{bc}\rangle \otimes |\Xi'_{\beta\gamma}\rangle, \tag{1.4}$$

solves equation

$$(c(i)e^{-\phi(i)} - c(-i)e^{-\phi(-i)})|\Xi\rangle + |\Xi\rangle * |\Xi\rangle = 0. \tag{1.5}$$

Equation (1.5) defines the ghost sliver in the $-1$ picture and appears after a special rescaling of super VSFT(VSSFT).

The paper is organized as follows. In section 2 we give an algebraic construction of the fermionic ghost sliver and wedge states in the minus one picture. In Section 3 following [3] we introduce the twisted $\beta$, $\gamma$ CFT and find ghost slivers in pictures -1 and 0. Further we use the twisted CFT to show that the sliver (1.4) satisfies equation (1.5). In section 4 it is shown that this equation arises as one of the equations of motion for super VSFT action.

# 2 Algebraic Construction.

In this section we will construct algebraically the $\beta, \gamma$ ghost sliver in the $-1$ picture. It is convenient to start from this picture because in this picture $\beta$,$\gamma$ annihilation and creation operators are defined symmetrically. It will be shown that the matrix of this sliver coincides with the one of the fermionic matter sliver.

## 2.1 Algebraic construction of sliver

The fermionic ghosts have the following mode expansion

$$\beta_{\pm}(\sigma) = \sum_r \beta_r e^{\pm ir\sigma}, \quad \gamma_{\pm}(\sigma) = \sum_r \gamma_r e^{\pm ir\sigma}, \tag{2.1}$$

where $r \in \mathbb{Z} + 1/2$, $\sigma \in [0, \pi]$. The modes $\beta_r$, $\gamma_r$ have the following commutation relations

$$[\gamma_r, \beta_s] = \delta_{r+s,0}. \tag{2.2}$$

Ghosts $\beta(\sigma)$ and $\gamma(\sigma)$ have weights $3/2$ and $-1/2$ correspondingly. Representation of the commutation relations (2.2) is specified by a chosen picture. $|q\rangle$ is a vacuum in the $q$ picture which is defined as

$$\beta_s|q\rangle = 0, \quad s > -q - 3/2, \tag{2.3a}$$
$$\gamma_r|q\rangle = 0, \quad r \geq q + 3/2. \tag{2.3b}$$

The adjoint of the $q$-vacuum is a state $\langle -q-Q|$ such that $\langle -q-Q|q\rangle = 1$ with $Q=2$. A distinguished feature of the -1 picture is that the adjoint vacuum has the same $q$-charge. Due to this property in the -1 picture one can define superghost state multiplication without any additional insertions

$$(|A\rangle *'|B\rangle)_3 = {}_1\langle A|_2\langle B|V^{\beta\gamma}\rangle_{123}, \tag{2.4}$$

where $|V^{\beta\gamma}\rangle$ is the three string vertex in the minus one picture. An algebraic construction of the fermionic ghost vertices: identity, reflector and three string vertex [55] over the vacuum in the $-1$ picture is reviewed in Appendix A. We will use these vertices in our construction of the sliver in the $-1$ picture.

As in the NS matter case instead of finding a solution to projector equation

$$|\Xi^{\beta\gamma}\rangle *'|\Xi^{\beta\gamma}\rangle = |\Xi^{\beta\gamma}\rangle, \tag{2.5}$$

we will first construct a solution to the linear equation

$$|\Xi^{\beta\gamma}\rangle *'|-1\rangle = |\Xi^{\beta\gamma}\rangle. \tag{2.6}$$

As in the matter fermionic case equations (2.6) and (2.5) have the same nontrivial solutions.

We look for the $\beta$,$\gamma$ ghost sliver in the following squeezed form

$$|\Xi^{\beta\gamma}\rangle = \mathcal{N}^{\beta\gamma}\exp(\beta_{-r}S^{\beta\gamma}_{rs}\gamma_{-s})|-1\rangle, \tag{2.7}$$

where $\mathcal{N}^{\beta\gamma}$ is a normalization constant and $|-1\rangle$ is a vacuum in the fermionic ghost sector in the $-1$ picture. The following formulae for the squeezed states multiplication for fermionic ghosts holds

$$\begin{aligned}\langle -1|\exp(-\beta_r S_{rs}\gamma_s)\exp(\mu_r\beta_{-r}+\nu_r\gamma_{-r}+\beta_{-r}V_{rs}\gamma_{-s})|-1\rangle \\ = \det(1-S_{rl}V_{ls})^{-1}\exp(\nu_r(1-S_{rl}V_{lk})^{-1}S_{ks}\mu_s).\end{aligned} \tag{2.8}$$

Using reflector (A.10) one gets the BPZ conjugated state $\langle\Xi^{\beta\gamma}|$ of $|\Xi^{\beta\gamma}\rangle$

$$\langle\Xi^{\beta\gamma}| = \mathcal{N}^{\beta\gamma}\langle -1|\exp(-\beta_r(CS^{\beta\gamma}C)_{rs}\gamma_s). \tag{2.9}$$

Equation (2.6) can be rewritten in the form

$${}_1\langle\Xi^{\beta\gamma}|_2\langle -1|V^{\beta\gamma}\rangle_{123} = |\Xi^{\beta\gamma}\rangle_3. \tag{2.10}$$

Using (2.8) one gets from (2.10) the following equation

$$M^{\beta\gamma}_{21}(1-T^{\beta\gamma}M^{\beta\gamma}_{11})^{-1}T^{\beta\gamma}M^{\beta\gamma}_{12} + M^{\beta\gamma}_{11} = T^{\beta\gamma}. \tag{2.11}$$

Here we denote $T^{\beta\gamma} = CS^{\beta\gamma}$, where $C_{rs} = (-1)^r \delta_{rs}$. Assuming the following commutation relations

$$[T^{\beta\gamma}, M^{\beta\gamma}_{ab}] = 0, \quad \forall\ a, b, \tag{2.12}$$

and using the properties (A.22), equation (2.11) can be rewritten as

$$T^{\beta\gamma\,2} M^{\beta\gamma}_{11} - T^{\beta\gamma}(1 + CI^{\beta\gamma\,-1} M^{\beta\gamma}_{11}) + M^{\beta\gamma}_{11} = 0. \tag{2.13}$$

Explicit solutions to this equation are

$$T^{\beta\gamma}_{\pm} = \frac{1 + CI^{\beta\gamma\,-1} M^{\beta\gamma}_{11} \mp \sqrt{(1 + CI^{\beta\gamma\,-1} M^{\beta\gamma}_{11})^2 - 4M^{\beta\gamma\,2}_{11}}}{2M^{\beta\gamma}_{11}}. \tag{2.14}$$

The projector equation

$$_1\langle \Xi^{\beta\gamma}|_2\langle \Xi^{\beta\gamma}|V^{\beta\gamma}\rangle_{123} = |\Xi^{\beta\gamma}\rangle_3$$

together with (2.8) gives the following equation

$$(M^{\beta\gamma}_{12}, M^{\beta\gamma}_{21}) \left(1 - T^{\beta\gamma} \begin{pmatrix} M^{\beta\gamma}_{11} & M^{\beta\gamma}_{12} \\ M^{\beta\gamma}_{21} & M^{\beta\gamma}_{22} \end{pmatrix}\right)^{-1} \begin{pmatrix} T^{\beta\gamma} M^{\beta\gamma}_{21} \\ T^{\beta\gamma} M^{\beta\gamma}_{12} \end{pmatrix} + M^{\beta\gamma}_{11} = T^{\beta\gamma}. \tag{2.15}$$

This equation can be rewritten in the form

$$(T^{\beta\gamma} - CI^{\beta\gamma})(T^{\beta\gamma\,2} M^{\beta\gamma}_{11} - T^{\beta\gamma}(1 + CI^{\beta\gamma\,-1} M^{\beta\gamma}_{11}) + M^{\beta\gamma}_{11}) = 0. \tag{2.16}$$

One of the solutions to this equation is the identity and the other ones, as has been mentioned above, coincide with solutions of (2.14). Expressions (2.14) can be rewritten in terms of the matrices $F$, $\tilde{F}$ (see Appendix A) and $C$ as

$$T^{\beta\gamma}_{\pm} = -\frac{1}{F}(C\tilde{F} \pm i). \tag{2.17}$$

So we obtain that the matrix $S^{\beta\gamma} = -CT^{\beta\gamma}$ is identical to that of matter fermions. This is not surprising and has an origin in the fact that the fermionic ghost vertices can be obtained from the fermionic matter vertices by changing the sings of $F$, $\tilde{F}$ and $C$. Moreover the equation for the fermionic ghost sliver is identical to that for fermionic matter sliver.

## 2.2 Algebraic construction of wedge states

Here we give an algebraic construction of the fermionic ghost wedge states in the minus one picture following steps of [13] for the bosonic case. The algebra obeyed by the wedge states is

$$|n\rangle *' |m\rangle = |n + m - 1\rangle. \tag{2.18}$$

We look for the ghost wedge states of the following form

$$|n\rangle = \mathcal{N}_n^{\beta\gamma} \exp(-\beta_{-r}(CT_n^{\beta\gamma})_{rs}\gamma_{-s})|-1\rangle. \tag{2.19}$$

The recursion for the wedge states exponential factors and norms can be explicitly written using the relation

$$|n\rangle *' |2\rangle = |n+1\rangle, \tag{2.20}$$

where $|2\rangle$ corresponds to the vacuum in the minus one picture. For the ghost wedges one gets

$$T_{n+1}^{\beta\gamma} = \frac{M_{11}^{\beta\gamma}(1 - CI^{\beta\gamma -1}T_n^{\beta\gamma})}{1 - T_n^{\beta\gamma}M_{11}^{\beta\gamma}}, \tag{2.21}$$

$$\mathcal{N}_{n+1}^{\beta\gamma} = \mathcal{N}_n^{\beta\gamma} \det\left(\frac{1 - M_{11}^{\beta\gamma}T_n^{\beta\gamma}}{1 - M_{11}^{\beta\gamma}T^{\beta\gamma}}\right)^{-1}. \tag{2.22}$$

Using

$$M_{11}^{\beta\gamma} = \frac{T^{\beta\gamma}}{1 - CI^{\beta\gamma -1}T^{\beta\gamma} + T^{\beta\gamma\, 2}} = \frac{(CI^{\beta\gamma})^5 - CI^{\beta\gamma}}{(1 - (CI^{\beta\gamma})^2)(1 - 3(CI^{\beta\gamma})^2)} \tag{2.23}$$

one finds

$$T_n^{\beta\gamma} = T^{\beta\gamma}\frac{1 - \Upsilon^{n-2}}{T^{\beta\gamma\, 2} - \Upsilon^{n-2}}, \tag{2.24}$$

$$\mathcal{N}_n^{\beta\gamma} = \det\left(\Upsilon^{-n+2}\frac{-T^{\beta\gamma}}{T_n^{\beta\gamma} - T^{\beta\gamma}}\right)^{-1} = \det\left(\frac{1 - T^{\beta\gamma\, 2}\Upsilon^{-n+2}}{1 - T^{\beta\gamma\, 2}}\right)^{-1}, \tag{2.25}$$

where

$$\Upsilon = -\frac{(CI^{\beta\gamma} - T^{\beta\gamma})}{T^{\beta\gamma}(1 - CI^{\beta\gamma}T^{\beta\gamma})}. \tag{2.26}$$

For the fermionic matter wedges

$$|n\rangle_m = \mathcal{N}_n^{10} \exp(-\frac{1}{2}\psi^\dagger CT_n\psi^\dagger)|0\rangle, \tag{2.27}$$

we get the same equations, but the matrices $F$, $\tilde{F}$, $C$ should be interchanged with $-F$, $-\tilde{F}$, $-C$ and the power in norm should be changed from $-1$ to $1/2$.

# 3 Conformal Construction.

In this section we present the twisted superghost conformal theory and derive corresponding equations in analogy with the one constructed by Gaiotto, Rastelli, Sen and Zwiebach [3].

## 3.1 Twisted CFT

A twisted CFT is introduced by subtracting from the stress energy tensor $T(w)$ of the $(\beta, \gamma)$ system the derivative of $U(1)$ ghost number current $j$ as follows

$$T'(w) = T(w) - \partial j(w), \quad \bar{T}'(\bar{w}) = \bar{T}(\bar{w}) - \bar{\partial}\bar{j}(\bar{w}), \quad j = -\beta\gamma. \tag{3.1}$$

More explicitly for the holomorphic stress energy tensor one obtains

$$T(w) = -\frac{3}{2}\beta\partial\gamma(w) - \frac{1}{2}\partial\beta\gamma(w), \quad \text{with } c = 11, \tag{3.2}$$

$$T'(w) = -\frac{1}{2}\beta'\partial\gamma'(w) + \frac{1}{2}\partial\beta'\gamma'(w), \quad \text{with } c = -1, \tag{3.3}$$

where $(\beta', \gamma')$ denotes the superghosts of the twisted CFT and $c$ is the central charge. Due to this modification the weights of the $\beta'$ and $\gamma'$ become equal to $1/2$

$$T'(w)\beta'(z) = \frac{1}{2}\frac{\beta'(w)}{(z-w)^2} + \frac{\partial\beta'(w)}{z-w}, \tag{3.4a}$$

$$T'(w)\gamma'(z) = \frac{1}{2}\frac{\gamma'(w)}{(z-w)^2} + \frac{\partial\gamma'(w)}{z-w}, \tag{3.4b}$$

and the superghost current $j' = -\beta'\gamma'$ has no anomaly. Fermionic ghosts in the original theory are bosonised as

$$\gamma(w) = \eta e^{\phi}(w), \quad \beta(w) = e^{-\phi}\partial\xi(w), \tag{3.5}$$

so that the ghost number current is expressed in the form $j = -\partial\phi$. The Euclidean world-sheet actions $S$ and $S'$ for the fields $\phi$ and $\phi'$ correspondingly are related as

$$S[\phi] = S'[\phi] - \frac{1}{2\pi}\int_{\Sigma} d^2\zeta\ \sqrt{g}R^{(2)}(\phi + \bar{\phi}), \tag{3.6}$$

where $\zeta$ denotes the world-sheet coordinates, $g$ denotes the Euclidean world-sheet metric and $R^{(2)}$ is the scalar curvature.

We assume that scalar curvature is proportional to $\delta$-function, which has a support on the infinity in some coordinates on $\Sigma$. Therefore we can identify the fields $\phi$ and $\phi'$ of two CFTs. The states in the two theories can be identified by the following map between the oscillators and the vacuum states

$$\beta_n \leftrightarrow \beta'_n, \quad \gamma_n \leftrightarrow \gamma'_n, \quad |-1\rangle \leftrightarrow |0'\rangle, \quad \langle -1| \leftrightarrow \langle 0'|, \quad \langle 0'|0'\rangle = 1, \tag{3.7}$$

where $|0\rangle$ and $|0'\rangle$ are the $SL(2,\mathbb{R})$ invariant vacua of two theories and $|-1\rangle = e^{-\phi(0)}|0\rangle$.

In the CFT$'$ the fields $\beta', \gamma'$ are bosonized as in the original theory

$$\gamma'(w) = \eta e^{\phi}(w), \quad \beta'(w) = e^{-\phi}\partial\xi(w). \tag{3.8}$$

Notice that we do not introduce new notations for the $(\xi, \eta)$ system because it has not changed.

One gets the following operator product expansions

$$T'(z)e^{\pm\phi}(w) = -\frac{1}{2}\frac{e^{\pm\phi}}{(z-w)^2} + \frac{\partial e^{\pm\phi}}{z-w}, \tag{3.9a}$$

$$T'(z)e^{\pm\phi/2}(w) = -\frac{1}{8}\frac{e^{\pm\phi/2}}{(z-w)^2} + \frac{\partial e^{\pm\phi/2}}{z-w}. \tag{3.9b}$$

## 3.2 Fermionic ghost surface states

In this subsection we construct superghost surface states using CFT methods. The advantage of the CFT method in comparison with the operator method, that we have used in Section 2, is that we do not have to postulate the sliver equation from the very beginning. The aim of this section is to define a sliver state as a surface state over $SL(2,\mathbb{R})$ invariant vacuum in CFT and CFT$'$, correspondingly, by the conformal map used in the matter case.

First we define the surface state for the original $(\beta, \gamma)$ system. The fermionic ghost surface state corresponding to the conformal map $\lambda(\xi)$ is defined as

$$\langle\Lambda| = \mathcal{N}_{\beta\gamma}\langle 0|\exp(-\sum_{\substack{r\geq 3/2\\ s\geq -1/2}}\gamma_r\Lambda_{rs}\beta_s), \tag{3.10}$$

where $\mathcal{N}_{\beta\gamma}$ is a normalization factor and the matrix $\Lambda_{rs}$ is defined so that the following identity holds

$$\langle 0|\exp(-\sum_{\substack{r\geq 3/2\\ s\geq -1/2}}\gamma_r\Lambda_{rs}\beta_s)\gamma(w)\beta(z)e^{-Q\phi(0)}|0\rangle = \langle\lambda\circ\gamma(w)\lambda\circ\beta(z)\lambda\circ e^{-Q\phi(0)}\rangle. \tag{3.11}$$

One can evaluate $\Lambda_{rs}$ explicitly. To this end one has to calculate the left hand side and right hand side of (3.11). Substitution of $\gamma(w) = \sum_r \gamma_{-r}w^{r+1/2}$ and $\beta(z) = \sum_s \beta_{-s}z^{s-3/2}$ into the left hand side of (3.11) yields

$$h(z,w) \equiv \langle 0|\exp(-\gamma_r\Lambda_{rs}\beta_s)\gamma(w)\beta(z)e^{-Q\phi(0)}|0\rangle = -\sum_{r,s} w^{r+1/2}z^{s-3/2}\Lambda_{rs}, \tag{3.12}$$

therefore

$$\Lambda_{rs} = -\oint\frac{dz}{2\pi i}\frac{1}{z^{r-1/2}}\oint\frac{dw}{2\pi i}\frac{1}{w^{s+3/2}}h(z,w). \tag{3.13}$$

Further one evaluates the correlation function in the right hand side of (3.11)

$$
\begin{aligned}
&\langle\lambda \circ \gamma(w)\lambda \circ \beta(z)\lambda \circ e^{-Q\phi(0)}\rangle \\
&\qquad = \langle\left(\frac{\partial\lambda(w)}{\partial w}\right)^{-1/2}\gamma(\lambda(w))\left(\frac{\partial\lambda(z)}{\partial z}\right)^{3/2}\beta(\lambda(z))e^{-Q\phi(\lambda(0))}\rangle \\
&\qquad = \left(\frac{\partial\lambda(w)}{\partial w}\right)^{-1/2}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{3/2}\langle\eta e^{\phi}(\lambda(w))e^{-\phi}\partial\xi(\lambda(z))e^{-Q\phi(\lambda(0))}\rangle \\
&\qquad = \left(\frac{\partial\lambda(w)}{\partial w}\right)^{-1/2}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{3/2}\frac{1}{\lambda(w)-\lambda(z)}\left(\frac{\lambda(w)-\lambda(0)}{\lambda(z)-\lambda(0)}\right)^{-Q}. \qquad (3.14)
\end{aligned}
$$

One gets the following answer

$$
\Lambda_{rs} = \oint\frac{dz}{2\pi i}\frac{1}{z^{r-1/2}}\oint\frac{dw}{2\pi i}\frac{1}{w^{s+3/2}}\left(\frac{\partial\lambda(w)}{\partial w}\right)^{-1/2}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{3/2}\frac{1}{\lambda(z)-\lambda(w)}\left(\frac{\lambda(w)-\lambda(0)}{\lambda(z)-\lambda(0)}\right)^{-2}. \qquad (3.15)
$$

The fermionic ghost surface state in CFT' corresponding to the conformal map $\lambda(\xi)$ is defined as

$$
\langle\Lambda'| = \mathcal{N}'_{\beta\gamma}\langle 0'|\exp(-\sum_{\substack{r\geq 1/2\\ s\geq 1/2}}\gamma_r\Lambda'_{rs}\beta_s), \qquad (3.16)
$$

where $\mathcal{N}'_{\beta\gamma}$ is a normalization factor and the matrix $\Lambda'_{rs}$ is defined so that the following identity holds

$$
\langle 0'|\exp(-\sum_{\substack{r\geq 1/2\\ s\geq 1/2}}\gamma_r\Lambda'_{rs}\beta_s)\gamma'(w)\beta'(z)|0'\rangle = \langle\lambda\circ\gamma'(w)\lambda\circ\beta'(z)\rangle'. \qquad (3.17)
$$

Substitution of $\gamma'(w) = \sum_r \gamma_{-r}w^{r-1/2}$ and $\beta'(z) = \sum_s \beta_{-s}z^{s-1/2}$ into the left hand side of (3.17) yields

$$
h'(z,w) \equiv \langle 0'|\exp(-\sum_{\substack{r\geq 1/2\\ s\geq 1/2}}\gamma_r\Lambda'_{rs}\beta_s)\gamma'(w)\beta'(z)|0'\rangle = -\sum_{r,s} w^{r-1/2}z^{s-1/2}\Lambda'_{rs}, \qquad (3.18)
$$

therefore

$$
\Lambda'_{rs} = -\oint\frac{dz}{2\pi i}\frac{1}{z^{r+1/2}}\oint\frac{dw}{2\pi i}\frac{1}{w^{s+1/2}}h'(z,w). \qquad (3.19)
$$

Evaluating the correlation function in the right hand side of (3.17) one finds

$$\langle\lambda\ \circ\ \gamma'(w)\lambda\ \circ\ \beta'(z)\rangle' = \langle\left(\frac{\partial\lambda(w)}{\partial w}\right)^{1/2}\gamma'(\lambda(w))\left(\frac{\partial\lambda(z)}{\partial z}\right)^{1/2}\beta'(\lambda(z))\rangle'$$
$$= \left(\frac{\partial\lambda(w)}{\partial w}\right)^{1/2}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{1/2}\langle\eta e^{\phi}(\lambda(w))e^{-\phi}\partial\xi(\lambda(z))\rangle' = \left(\frac{\partial\lambda(w)}{\partial w}\right)^{1/2}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{1/2}\frac{1}{\lambda(w)-\lambda(z)}. \tag{3.20}$$

One gets the following answer

$$\Lambda'_{rs} = \oint\frac{dz}{2\pi i}\frac{1}{z^{r+1/2}}\oint\frac{dw}{2\pi i}\frac{1}{w^{s+1/2}}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{1/2}\left(\frac{\partial\lambda(w)}{\partial w}\right)^{1/2}\frac{1}{\lambda(z)-\lambda(w)}. \tag{3.21}$$

It should be mentioned here that the matrix (3.21) coincides with the matrix of the matter sliver [39],[40]. This fact was also obtained using algebraic methods in the previous section.

### 3.3 Relationship between star products

Now we give a relationship between the star-products in the two theories. We denote these products by $*$ and $*'$ respectively.

$$\langle A|B*C\rangle = \langle f_1\circ A(0)f_2\circ B(0)f_3\circ C(0)\rangle, \tag{3.22a}$$
$$\langle A|B*'C\rangle = \langle f_1\circ A'(0)f_2\circ B'(0)f_3\circ C'(0)\rangle'. \tag{3.22b}$$

where $f_1(z)=h_2^{-1}(h_3(z))$, $f_2(z)=h_2^{-1}(e^{2\pi i/3}h_3(z))$, $f_3(z)=h_2^{-1}(e^{4\pi i/3}h_3(z))$ and

$$h_n(z) = \left(\frac{1+iz}{1-iz}\right)^{\frac{2}{n}} \tag{3.23}$$

are the standard conformal maps used in the definition of the star-product.

The actions of the two theories on a flat world-sheet with a single defect at the common midpoint of three strings where we have the deficit of angle of $-\pi$ are related as

$$S'[\phi] = S[\phi] - \frac{1}{2}(\phi(M)+\bar{\phi}(M)), \tag{3.24}$$

where $M$ denotes the location of the midpoint. Since the action appears in the path integral through the combination $e^{-S}$ we have

$$\langle f_1\circ A(0)f_2\circ B(0)f_3\circ C(0)\rangle = K_0\langle f_1\circ A'(0)f_2\circ B'(0)f_3\circ C'(0)\rho^{+\prime}(M)\rho^{-\prime}(M)\rangle', \tag{3.25}$$

where $K_0$ is some normalization constant,

$$\rho^{+\prime} = e^{-\phi/2}, \quad \rho^{-\prime} = e^{-\bar{\phi}/2}, \tag{3.26}$$

and $M = f_1(i) = f_2(i) = f_3(i)$. Since in the local coordinate system the mid-point of the string is at the point $i$, we get

$$f_1 \circ A'(0)\rho^{+\prime}(M)\rho^{-\prime}(M) = \lim_{\epsilon\to 0} |f_1'(i+\epsilon)|^{1/4} f_1 \circ (A'(0)\rho^{+\prime}(i+\epsilon)\rho^{-\prime}(i+\epsilon)). \tag{3.27}$$

Using the BPZ conjugation $I(z) = -1/z$

$$I \circ (\rho^{+\prime}(i+\epsilon)\rho^{-\prime}(i+\epsilon)) \simeq \rho^{+\prime}(i-\epsilon)\rho^{-\prime}(i-\epsilon) \tag{3.28}$$

we get

$$|B * C\rangle = \lim_{\epsilon\to 0} |f_1'(i+\epsilon)|^{1/4} \rho^{+\prime}(i-\epsilon)\rho^{-\prime}(i-\epsilon)|B *' C\rangle \propto \rho^{+\prime}(i-\epsilon)\rho^{-\prime}(i-\epsilon)|B *' C\rangle. \tag{3.29}$$

Here we omit the possible infinite scale factor since we are analyzing the solution up to the normalization.

## 3.4 Ghost sliver equation in twisted CFT

Here we show that the direct product of the twisted $(b,c)$ and twisted $(\beta,\gamma)$ slivers

$$|\Xi\rangle = |\Xi'_{bc}\rangle \otimes |\Xi'_{\beta\gamma}\rangle \tag{3.30}$$

solves the following equation

$$(c(i)e^{-\phi(i)} - c(-i)e^{-\phi(-i)})|\Xi\rangle + |\Xi * \Xi\rangle = 0. \tag{3.31}$$

Using the $(b,c)$ CFT′ [3] and $(\beta,\gamma)$ CFT′ one rewrites (3.31) as

$$\begin{aligned}
&(c(i)e^{-\phi(i)} - c(-i)e^{-\phi(-i)})|\Xi'_{bc}\rangle \otimes |\Xi'_{\beta\gamma}\rangle \\
&\qquad \propto -\sigma^{+\prime}(i-\epsilon)\sigma^{-\prime}(i-\epsilon)\rho^{+\prime}(i-\epsilon)\rho^{-\prime}(i-\epsilon)|\Xi'_{bc} *' \Xi'_{bc}\rangle \otimes |\Xi'_{\beta\gamma} *' \Xi'_{\beta\gamma}\rangle \\
&\qquad\qquad \propto -\sigma^{+\prime}(i-\epsilon)\sigma^{-\prime}(i-\epsilon)\rho^{+\prime}(i-\epsilon)\rho^{-\prime}(i-\epsilon)|\Xi'_{bc}\rangle \otimes |\Xi'_{\beta\gamma}\rangle,
\end{aligned} \tag{3.32}$$

where $\sigma^{+\prime} = e^{i\varphi/2}$, $\sigma^{-\prime} = e^{i\bar{\varphi}/2}$ are conformal operators of the weight $-1/8$ in the twisted CFT and bosonic ghosts are bosonized as

$$c(z) = e^{i\varphi}(z), \quad b(z) = e^{-i\varphi}(z). \tag{3.33}$$

Let us take the inner product of (3.32) with a Fock space state $\langle\Phi|$. Using definitions of the sliver and relation $c(\pm i) = \pm i c'(\pm i)$ one gets for the left hand side

$$\langle f\circ\left(\Phi'(0)(c'(i)e^{-\phi(i)} + c'(-i)e^{-\phi(-i)})\right)\rangle' = \langle f\circ\Phi'(0)\left(c'(i\infty)e^{-\phi(i\infty)} + c'(-i\infty)e^{-\phi(-i\infty)}\right)\rangle'. \tag{3.34}$$

The right hand side of (3.32) is proportional to

$$\begin{aligned}\langle f\circ\left(\Phi'(0)\sigma^{+\prime}(i+\epsilon)\sigma^{-\prime}(i+\epsilon)\rho^{+\prime}(i+\epsilon)\rho^{-\prime}(i+\epsilon)\right)\rangle' \\ \propto \langle f\circ\Phi'(0)\sigma^{+\prime}(i\eta)\sigma^{-\prime}(i\eta)\rho^{+\prime}(i\eta)\rho^{-\prime}(i\eta)\rangle' \\ \propto \langle f\circ\Phi'(0)\sigma^{+\prime}(i\eta)\sigma^{+\prime}(-i\eta)\rho^{+\prime}(i\eta)\rho^{+\prime}(-i\eta)\rangle',\end{aligned} \tag{3.35}$$

where $f(i+\epsilon)\simeq\frac{1}{2i}\ln\frac{i\epsilon}{2}\equiv i\eta$ and one has used the Neumann boundary conditions on $\varphi$ and $\phi$ to relate $\sigma^{-\prime}(i\eta)$ and $\sigma^{+\prime}(-i\eta)$ and $\rho^{-\prime}(i\eta)$ and $\rho^{+\prime}(-i\eta)$. Since both correlators are being evaluated on the upper half plane, the points $\pm i\infty$ correspond to the same points. The leading terms in the operator product expansion of $\sigma^{+\prime}(i\eta)$ with $\sigma^{+\prime}(-i\eta)$ and $\rho^{+\prime}(i\eta)$ with $\rho^{+\prime}(-i\eta)$ are $c(i\infty)$ and $e^{-\phi(i\infty)}$ correspondingly. So one gets that (3.30) solves the equation (3.31).

# 4 Vacuum Superstring Field Theory Equations.

In this section we will describe how the singular VSSFT action could arise. A shift of the SSFT action for both GSO+ and GSO− sectors to the tachyon vacuum yields the following action

$$\begin{aligned}S[\mathcal{A}_+,\mathcal{A}_-] = \frac{1}{g_0^2}\Big[\frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_+, Q_{\mathsf{odd}}\mathcal{A}_+\rangle\rangle + \frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_-, Q_{\mathsf{odd}}\mathcal{A}_-\rangle\rangle - \langle\langle Y_{-2}|\mathcal{A}_+, Q_{\mathsf{even}}\mathcal{A}_-\rangle\rangle \\ +\frac{1}{3}\langle\langle Y_{-2}|\mathcal{A}_+,\mathcal{A}_+,\mathcal{A}_+\rangle\rangle - \langle\langle Y_{-2}|\mathcal{A}_+,\mathcal{A}_-,\mathcal{A}_-\rangle\rangle\Big],\end{aligned} \tag{4.1}$$

where $g_0$ is an open string coupling constant, $Y_{-2} = Y(i)Y(-i)$, $Y(z) = 4c\partial\xi e^{-2\phi}(z)$ is a double step inverse picture changing operator and $Q_{\mathsf{odd}}$ and $Q_{\mathsf{even}}$ are

$$Q_{\mathsf{odd}}Z = Q_B Z + A_{0,+} * Z - (-1)^{|Z|} Z * A_{0,+}, \tag{4.2a}$$

$$Q_{\mathsf{even}}Z = A_{0,-} * Z + (-1)^{|Z|} Z * A_{0,-}, \tag{4.2b}$$

where $Z$ is a string field in GSO+ or GSO− sector, $|Z|$ is a parity of the field $Z$ and $Q_B$ is the BRST charge of superstrings. $Q_{\mathsf{odd}}$ and $Q_{\mathsf{even}}$ are expected to be regular since the solution $A_{0,+}$, $A_{0,-}$ describing the tachyon vacuum is regular. Moreover $Q_{\mathsf{odd}}$ and $Q_{\mathsf{even}}$ satisfy the set of axioms [41]. On the other hand it is known that the VSSFT action is singular. The mechanism describing how this singularity could arise was proposed by Gaiotto, Rastelli, Sen and Zwiebach [3]. We will describe it below for the case of the action (4.1). OSSFT and VSSFT are related via a singular field redefinition. Let us begin with $Q_{\mathsf{odd}}$ and $Q_{\mathsf{even}}$ of the form

$$Q_{\mathsf{odd}} = \sum_r \int d\sigma\ a_r(\sigma) A_r(\sigma), \tag{4.3a}$$

$$Q_{\mathsf{even}} = \sum_r \int d\sigma\ b_r(\sigma) B_r(\sigma), \tag{4.3b}$$

where $a_r$ and $b_r$ are forms of degree $1-h_r$ and $A_r$ and $B_r$ are correspondingly Grassmann odd and even local operators of superghost number one. We use a double trick, so that $\sigma$ runs from $-\pi$ to $\pi$ and we have only holomorphic fields. We fix the coordinate system on local patches. Since $A_{0,+}$ and $A_{0,-}$ are regular we expect $a_r(\sigma)$ and $b_r(\sigma)$ to be smooth functions of $\sigma$ in local patches [3].

We reparametrize open string coordinate $\sigma$ to $f(\sigma)$ so that $f(\pi-\sigma)=\pi-f(\sigma)$ for $0\leq\sigma\leq\pi$ and $f(-\sigma-\pi)=-\pi-f(\sigma)$ for $-\pi\leq\sigma\leq 0$. Such reparametrization preserves the star-product, but transforms $Q_{\mathsf{odd}}$ and $Q_{\mathsf{even}}$ to

$$Q_{\mathsf{odd}} = \sum_r \int d\sigma\ a_r(\sigma)(f'(\sigma))^{h_r} A'_r(f(\sigma)), \tag{4.4a}$$

$$Q_{\mathsf{even}} = \sum_r \int d\sigma\ b_r(\sigma)(f'(\sigma))^{h_r} B'_r(f(\sigma)). \tag{4.4b}$$

Consider $f(\sigma)$ squeezed near the midpoint so that $f'(\pm\pi/2)$ is small and $\int d\sigma\ (f'(\sigma))^{h_r}$, $h_r<0$ gets a large contribution from a region around the midpoint. For example one may choose $f'(\sigma)\simeq(\sigma\mp\pi/2)^2+\epsilon^2$ for $\sigma\simeq\pm\pi/2$. One gets that $Q_{\mathsf{odd}}$ and $Q_{\mathsf{even}}$ get a dominant contribution from the lowest dimension operators $c$ and $\gamma$

$$Q_{\mathsf{odd}} = \epsilon^{-1}(c(i)-c(-i)), \tag{4.5a}$$

$$Q_{\mathsf{even}} = \epsilon^{-1/2}(\gamma(i)-\gamma(-i)), \tag{4.5b}$$

Now we could make a singular field redefinition $\mathcal{A}_+=\epsilon^{-1}\mathrm{A}_+$, $Q_{\mathsf{odd}}=\epsilon^{-1}\mathcal{Q}_{\mathsf{odd}}$, $\mathcal{A}_-=\epsilon^{-1}\mathrm{A}_-$ and $Q_{\mathsf{even}}=\epsilon^{-1}\mathcal{Q}_{\mathsf{even}}$. Since $\mathcal{Q}_{\mathsf{odd}}=c(i)-c(-i)$ and $\mathcal{Q}_{\mathsf{even}}=\epsilon^{1/2}(\gamma(i)-\gamma(-i))$ this choice of kinetic operators satisfies axioms [41]. Actually we get that $\mathcal{Q}_{\mathsf{even}}\to 0$ as $\epsilon\to 0$. The action (4.1) takes the form

$$S[\mathrm{A}_+,\mathrm{A}_-] = \frac{\kappa_0}{g_0^2}\left[\frac{1}{2}\langle\langle Y_{-2}|\mathrm{A}_+,\mathcal{Q}_{\mathsf{odd}}\mathrm{A}_+\rangle\rangle+\frac{1}{2}\langle\langle Y_{-2}|\mathrm{A}_-,\mathcal{Q}_{\mathsf{odd}}\mathrm{A}_-\rangle\rangle\right.$$
$$\left.+\frac{1}{3}\langle\langle Y_{-2}|\mathrm{A}_+,\mathrm{A}_+,\mathrm{A}_+\rangle\rangle-\langle\langle Y_{-2}|\mathcal{A}_+,\mathrm{A}_-,\mathrm{A}_-\rangle\rangle\right], \tag{4.6}$$

where $\kappa_0 = \epsilon^{-3}$. The equations of motion for this action are

$$\mathcal{Q}_{\mathsf{odd}} \mathrm{A}_+ + \mathrm{A}_+ * \mathrm{A}_+ - \mathrm{A}_- * \mathrm{A}_- = 0, \tag{4.7a}$$

$$\mathcal{Q}_{\mathsf{odd}} \mathrm{A}_- + \mathrm{A}_+ * \mathrm{A}_- - \mathrm{A}_- * \mathrm{A}_+ = 0. \tag{4.7b}$$

However to get non-trivial solution for $\mathrm{A}_-$ it is convenient to make another field redefinition: $\mathcal{A}_+ = \epsilon^{-1}\mathrm{A}_+$, $Q_{\mathsf{odd}} = \epsilon^{-1}\mathcal{Q}_{\mathsf{odd}}$, $\mathcal{A}_- = \epsilon^{-1/2}\mathrm{A}'_-$ and $Q_{\mathsf{even}} = \epsilon^{-1/2}\mathcal{Q}'_{\mathsf{even}}$. One gets the following singular action

$$\begin{aligned} S[\mathrm{A}_+, \mathrm{A}'_-] = {} & \frac{\kappa_0}{g_0^2}\left[\frac{1}{2}\langle\langle Y_{-2}|\mathrm{A}_+, \mathcal{Q}_{\mathsf{odd}}\mathrm{A}_+\rangle\rangle + \frac{1}{3}\langle\langle Y_{-2}|\mathrm{A}_+, \mathrm{A}_+, \mathrm{A}_+\rangle\rangle\right] \\ & + \frac{\kappa'_0}{g_0^2}\left[\frac{1}{2}\langle\langle Y_{-2}|\mathrm{A}'_-, \mathcal{Q}_{\mathsf{odd}}\mathrm{A}'_-\rangle\rangle - \langle\langle Y_{-2}|\mathrm{A}_+, \mathcal{Q}'_{\mathsf{even}}\mathrm{A}'_-\rangle\rangle - \langle\langle Y_{-2}|\mathrm{A}_+, \mathrm{A}'_-, \mathrm{A}'_-\rangle\rangle\right], \end{aligned} \tag{4.8}$$

where $\kappa'_0 = \epsilon^{-2}$.

The gauge invariance in terms of redefined fields and charges is

$$\delta\mathrm{A}_+ = \mathcal{Q}_{\mathsf{odd}}\Lambda_+ + [\mathrm{A}_+, \Lambda_+] + \epsilon^{1/2}(\mathcal{Q}'_{\mathsf{even}}\Lambda_- + [\mathrm{A}'_-, \Lambda_-]), \tag{4.9a}$$

$$\delta\mathrm{A}'_- = \mathcal{Q}'_{\mathsf{even}}\Lambda_+ + [\mathrm{A}'_-, \Lambda_+] + \epsilon^{-1/2}(\mathcal{Q}_{\mathsf{odd}}\Lambda_- + [\mathrm{A}_+, \Lambda_-]). \tag{4.9b}$$

This action gives the following equations of motion

$$\mathcal{Q}_{odd}\mathrm{A}_+ + \mathrm{A}_+ * \mathrm{A}_+ - \epsilon^{-1}(\mathcal{Q}'_{even}\mathrm{A}'_- + \mathrm{A}'_- * \mathrm{A}'_-) = 0, \tag{4.10a}$$

$$\mathcal{Q}_{odd}\mathrm{A}'_- + \mathrm{A}_+ * \mathrm{A}'_- - \mathcal{Q}'_{even}\mathrm{A}_+ - \mathrm{A}'_- * \mathrm{A}_+ = 0. \tag{4.10b}$$

Here we omit $Y_{-2}$. We will solve them in the factorized form

$$\mathcal{Q}_{odd}\mathrm{A}_+ + \mathrm{A}_+ * \mathrm{A}_+ = 0, \tag{4.11a}$$

$$\mathcal{Q}'_{even}\mathrm{A}'_- + \mathrm{A}'_- * \mathrm{A}'_- = 0, \tag{4.11b}$$

$$\mathcal{Q}_{odd}\mathrm{A}'_- + \mathrm{A}_+ * \mathrm{A}'_- = 0, \tag{4.11c}$$

$$\mathcal{Q}'_{even}\mathrm{A}_+ + \mathrm{A}'_- * \mathrm{A}_+ = 0. \tag{4.11d}$$

Equation (4.11a) is the equation for the twisted $(b, c)$ sliver. The fermionic ghost part of $\mathrm{A}_+$ should be chosen as a conformal sliver, i.e. the sliver in the zero picture, that satisfies the projector equation.

## 4.1 Ghost sliver equations

The surface state corresponding to the conformal map $f(\xi)$ with insertion $\phi_d(z)$ [2],[29] is defined as

$$U_f^\dagger \phi_d(z)|0\rangle, \tag{4.12}$$

where $f \circ \phi_d(z) \equiv [f'(z)]^d \phi'_d(f(z)) = U_f \phi_d(z) U_f^{-1}$ for the field $\phi_d(z)$ of the weight $d$ and $U_f^\dagger = U_{I \circ f \circ I}^{-1}$.

One obtains

$$U_f^\dagger I \circ f \circ I \circ \phi_d(z)|0\rangle = \phi_d(z) U_f^\dagger |0\rangle. \tag{4.13}$$

In particular for the map $f(\xi) = \arctan \xi$ corresponding to the sliver and the field $\phi_d(z)$ of the weight $d = 0$ one gets for $z = \pm i \pm \epsilon$, $\epsilon \to 0$

$$I \circ f \circ I \circ \phi_0(\pm i \mp \epsilon) = \phi_0(\pm i \eta^{-1}), \tag{4.14}$$

since

$$f(\pm i \pm \epsilon) = \arctan(\pm i \pm \epsilon) = \frac{\pm 1}{2i} \ln \frac{i\epsilon}{2} = \pm i\eta, \quad \eta \to \infty. \tag{4.15}$$

Let us consider equation (4.11b)

$$Y(i)Y(-i)[(\gamma(i) - \gamma(-i))\mathrm{A}'_- + \mathrm{A}'_- * \mathrm{A}'_-] = 0. \tag{4.16}$$

Let us choose $\mathrm{A}'_-$ as a sliver with insertion $\gamma(0)$

$$\mathrm{A}'_- = U_f^\dagger \gamma(0)|0\rangle. \tag{4.17}$$

Changing the picture one finds

$$Y(\pm i)\mathrm{A}'_- = U_f^\dagger Y(0)\gamma(0)|0\rangle = -4\, U_f^\dagger c(0) e^{-\phi(0)}|0\rangle \equiv \mathrm{A}_-^-. \tag{4.18}$$

Ghost part of $\mathrm{A}_-^-$ is direct product of the sliver in the minus one picture for the fermionic ghosts and the twisted sliver for the bosonic ghosts. So we get the following equation

$$(c(i)e^{-\phi(i)} - c(-i)e^{-\phi(-i)})\mathrm{A}_-^- + \mathrm{A}_-^- * \mathrm{A}_-^- = 0 \tag{4.19}$$

This is indeed the equation for the ghost sliver that we have solved in Section 3. Equations (4.11c) and (4.11d) are also satisfied for such $\mathrm{A}_+$ and $\mathrm{A}'_-$.

# 5 Conclusion and Discussions.

In this paper Neveu-Schwarz ghost slivers in pictures zero and minus one are constructed. It is shown that the sliver in a twisted CFT is in fact a projector with respect to the twisted star product and that this sliver multiplied by $(b, c)$ twisted sliver and other slivers solve super VSFT equations.

It is worth to note that our results can be used in the following directions. It is interesting to consider excitations on a solution of (1.5) to check if one can reproduce the standard perturbative spectrum of NS string. In spite of the fact that we consider only cubic superstring field theory, our results can be applied to the Berkovits non-polynomial superstring field theory.

Let us note that besides operator and CFT formalism string star algebra admits a nice half-string description [6, 7, 8, 42, 43, 44] (see [49] for a review and refs. therein) and it would be interesting to generalize this formalizm to superstrings.

One of the unsolved interesting problems is the construction of the analog of the continuous Moyal product for the fermionic star algebra in the framework of [44].

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# Appendix

## A $\beta,\gamma$ vertices.

In this section we give an algebraic construction of the fermionic ghost vertices: identity, reflector and three string vertex [55] over the vacuum in the $-1$ picture. Let us first solve the overlap equations for the identity state

$$[\beta_\pm(\sigma)\pm i\beta_\pm(\pi-\sigma)]|I^{\beta\gamma}\rangle=0,\quad \sigma\in(0,\pi/2), \tag{A.1a}$$
$$[\gamma_\pm(\sigma)\pm i\gamma_\pm(\pi-\sigma)]|I^{\beta\gamma}\rangle=0,\quad \sigma\in(0,\pi/2). \tag{A.1b}$$

Using the identity one can rewrite these overlaps in terms of $\beta_+(\sigma)$ and $\gamma_+(\sigma)$, which are defined on the whole interval $(-\pi,\pi)$

$$\beta_+(\sigma)=\beta_-(-\sigma),\quad \gamma_+(\sigma)=\gamma_-(-\sigma),\quad \sigma\in[-\pi,0], \tag{A.2}$$

in the following form

$$\beta_+(\sigma) = \begin{cases} -i\beta_+(\pi-\sigma), & (0,\pi/2), \\ i\beta_+(-\pi-\sigma), & (-\pi/2,0), \\ i\beta_+(\pi-\sigma), & (\pi/2,\pi), \\ -i\beta_+(-\pi-\sigma), & (-\pi,-\pi/2); \end{cases} \tag{A.3a}$$

$$\gamma_+(\sigma) = \begin{cases} -i\gamma_+(\pi-\sigma), & (0,\pi/2), \\ i\gamma_+(-\pi-\sigma), & (-\pi/2,0), \\ i\gamma_+(\pi-\sigma), & (\pi/2,\pi), \\ -i\gamma_+(-\pi-\sigma), & (-\pi,-\pi/2). \end{cases} \tag{A.3b}$$

Integration of these equations via the relations $\frac{1}{2\pi}\int_{-\pi}^{\pi} d\sigma e^{-ir\sigma}\beta_+(\sigma) = \beta_r$ and $\frac{1}{2\pi}\int_{-\pi}^{\pi} d\sigma e^{-ir\sigma}\gamma_+(\sigma) = \gamma_r$ yields the following overlap equations for the identity state

$$\beta_r = -F_{rs}\beta_s - \tilde{F}_{rs}\beta_{-s}, \tag{A.4a}$$
$$\beta_{-r} = \tilde{F}_{rs}\beta_s + F_{rs}\beta_{-s}, \tag{A.4b}$$
$$\gamma_r = -F_{rs}\gamma_s - \tilde{F}_{rs}\gamma_{-s}, \tag{A.4c}$$
$$\gamma_{-r} = \tilde{F}_{rs}\gamma_s + F_{rs}\gamma_{-s}. \tag{A.4d}$$

Here $r,s \geq \frac{1}{2}$ and the hermitian matrices $F_{rs}$ and $\tilde{F}_{rs}$ are the same as for the matter fermionic case [55]

$$F_{rs} = -\frac{2}{\pi}\frac{\imath^{r-s}}{r+s}, \quad r = s \bmod(2), \tag{A.5a}$$
$$\tilde{F}_{rs} = \frac{2}{\pi}\frac{\imath^{r+s}}{s-r}, \quad r = s+1 \bmod(2). \tag{A.5b}$$

They have the following properties

$$F^2 - \tilde{F}^2 = 1, \quad [F,\tilde{F}] = 0, \tag{A.6}$$
$$CFC = -F, \quad F^T = F, \quad C\tilde{F}C = \tilde{F}, \quad \tilde{F}^T = -\tilde{F}, \tag{A.7}$$

where $C_{rs} = (-1)^r\delta_{rs}$. It is reasonable to search for the identity state in the following form

$$|I^{\beta\gamma}\rangle = \exp(\beta_{-r}I^{\beta\gamma}_{rs}\gamma_{-s})|-1\rangle, \tag{A.8}$$

where $|-1\rangle$ is the vacuum in the minus one picture, which is annihilated by $\beta_r$, $\gamma_r$ for $r \geq 1/2$. One gets the following expressions for the matrix $I^{\beta\gamma}_{rs}$

$$I^{\beta\gamma} = -\frac{\tilde{F}}{1+F} = \frac{1-F}{\tilde{F}} = I^{-1}, \tag{A.9}$$

where $I$ is the identity of the matter fermions. Expression for reflector is found to be

$$|R^{\beta\gamma}\rangle = \exp(i\beta^1_{-r}(-1)^r\gamma^2_{-r} - i\beta^2_{-r}(-1)^r\gamma^1_{-r})|-1\rangle_{12}. \tag{A.10}$$

For the three string fermionic ghost vertex one gets the following overlap equations

$$[\beta^a_\pm(\sigma) \pm i\beta^{a-1}_\pm(\pi-\sigma)]|V^{\beta\gamma}\rangle = 0, \quad (0,\pi/2), \tag{A.11a}$$
$$[\gamma^a_\pm(\sigma) \pm i\gamma^{a-1}_\pm(\pi-\sigma)]|V^{\beta\gamma}\rangle = 0, \quad (0,\pi/2). \tag{A.11b}$$

These overlap equations can be explicitly solved in terms of the new $\mathbb{Z}_3$ Fourier variables

$$\mathcal{B}^1 = \frac{1}{\sqrt{3}}(\beta^1_+ + \beta^2_+ + \beta^3_+), \tag{A.12a}$$
$$\mathcal{B}^2 = \frac{1}{\sqrt{3}}(\beta^1_+ + \alpha\beta^2_+ + \alpha^*\beta^3_+) \equiv \mathcal{B}, \tag{A.12b}$$
$$\mathcal{B}^3 = \frac{1}{\sqrt{3}}(\beta^1_+ + \alpha^*\beta^2_+ + \alpha\beta^3_+) \equiv \bar{\mathcal{B}}; \tag{A.12c}$$
$$\mathcal{G}^1 = \frac{1}{\sqrt{3}}(\gamma^1_+ + \gamma^2_+ + \gamma^3_+), \tag{A.12d}$$
$$\mathcal{G}^2 = \frac{1}{\sqrt{3}}(\gamma^1_+ + \alpha\gamma^2_+ + \alpha^*\gamma^3_+) \equiv \mathcal{G}, \tag{A.12e}$$
$$\mathcal{G}^3 = \frac{1}{\sqrt{3}}(\gamma^1_+ + \alpha^*\gamma^2_+ + \alpha\gamma^3_+) \equiv \bar{\mathcal{G}}. \tag{A.12f}$$

In terms of these variables one gets the following overlaps

$$\mathcal{B}^1(\sigma) = \begin{cases} -\imath\mathcal{B}^1(\pi-\sigma), & |\sigma| \le \frac{\pi}{2}, \\ \imath\mathcal{B}^1(\pi-\sigma), & \frac{\pi}{2} \le |\sigma| \le \pi, \end{cases} \tag{A.13a}$$
$$\mathcal{B}^2(\sigma) = \begin{cases} -\imath\alpha\mathcal{B}^2(\pi-\sigma), & |\sigma| \le \frac{\pi}{2}, \\ \imath\alpha^*\mathcal{B}^2(\pi-\sigma), & \frac{\pi}{2} \le |\sigma| \le \pi, \end{cases} \tag{A.13b}$$
$$\mathcal{B}^3(\sigma) = \begin{cases} -\imath\alpha^*\mathcal{B}^3(\pi-\sigma), & |\sigma| \le \frac{\pi}{2}, \\ \imath\alpha\mathcal{B}^3(\pi-\sigma), & \frac{\pi}{2} \le |\sigma| \le \pi, \end{cases} \tag{A.13c}$$

and analogous ones for the variables $\mathcal{G}(\sigma)$. All these overlaps differ from the overlaps for the matter fermions only by the sign in the right hand side. The overlap equations for $\mathcal{B}(\sigma)$ can be rewritten in components as

$$\mathcal{B}_r = \frac{1}{2}F_{rs}\mathcal{B}_s + \frac{1}{2}(\tilde{F}_{rs} - \sqrt{3}C)\mathcal{B}_{-s}, \tag{A.14a}$$
$$\mathcal{B}_{-r} = -\frac{1}{2}(\tilde{F}_{rs} - \sqrt{3}C)\mathcal{B}_s - \frac{1}{2}F_{rs}\mathcal{B}_{-s}. \tag{A.14b}$$

and analogous for variables $\mathcal{G}$. We search for the three string vertex of the following form in terms of the new variables

$$|V^{\beta\gamma}\rangle = \exp(\mathcal{B}^1_{-r} I^{\beta\gamma}_{rs} \mathcal{G}^1_{-s} + \bar{\mathcal{B}}_{-r} U^{\beta\gamma}_{rs} \mathcal{G}_{-s} + \mathcal{B}_{-r} \tilde{U}^{\beta\gamma}_{rs} \bar{\mathcal{G}}_{-s})|-1\rangle_{123}. \tag{A.15}$$

By solving the overlaps for $\mathcal{B}$ and $\mathcal{G}$ one gets the following expressions for $U^{\beta\gamma}$ and $\tilde{U}^{\beta\gamma\, T}$

$$U^{\beta\gamma} = \tilde{U}^{\beta\gamma\, T} = \frac{\tilde{F} - \sqrt{3}C}{2-F} = U^{-1}, \tag{A.16a}$$

$$U^{\beta\gamma} = \tilde{U}^{\beta\gamma\, T} = -\frac{2+F}{\tilde{F} - \sqrt{3}C} = U^{-1}, \tag{A.16b}$$

where $U$ is the corresponding matrix for the matter fermions. Solving the overlaps for $\bar{\mathcal{B}}$ and $\bar{\mathcal{G}}$ one gets the following

$$U^{\beta\gamma\, T} = \tilde{U}^{\beta\gamma} = \frac{\tilde{F} + \sqrt{3}C}{2-F} = CU^{\beta\gamma}C, \tag{A.17a}$$

$$U^{\beta\gamma\, T} = \tilde{U}^{\beta\gamma} = -\frac{2+F}{\tilde{F} + \sqrt{3}C} = CU^{\beta\gamma}C. \tag{A.17b}$$

Rewriting the three string fermionic ghost vertex in terms of the old variables one gets

$$|V^{\beta\gamma}\rangle = \exp(\beta^a_{-r} V^{\beta\gamma,\, ab}_{rs} \gamma^b_{-s})|-1\rangle_{123}, \tag{A.18}$$

where

$$V^{\beta\gamma,\, a\, a} = \frac{1}{3}(I^{\beta\gamma} + U^{\beta\gamma} + CU^{\beta\gamma}C), \tag{A.19a}$$

$$V^{\beta\gamma,\, a\, a+1} = \frac{1}{3}(I^{\beta\gamma} + \alpha U^{\beta\gamma} + \alpha^* CU^{\beta\gamma}C), \tag{A.19b}$$

$$V^{\beta\gamma,\, a\, a-1} = \frac{1}{3}(I^{\beta\gamma} + \alpha^* U^{\beta\gamma} + \alpha CU^{\beta\gamma}C). \tag{A.19c}$$

More explicitly one obtains the following expressions

$$V^{\beta\gamma,\, 11} = \frac{F\tilde{F}}{(1+F)(2-F)}, \tag{A.20a}$$

$$V^{\beta\gamma,\, 12} = \frac{-\tilde{F} - iC(1+F)}{(1+F)(2-F)}, \tag{A.20b}$$

$$V^{\beta\gamma,\, 21} = \frac{-\tilde{F} + iC(1+F)}{(1+F)(2-F)}. \tag{A.20c}$$

We should mention here that the fermionic ghost vertices differs from that of the fermionic matter only by the signs of the matrices $F$, $\tilde{F}$, $C$. Changing the signs of these matrices one gets the matter fermionic vertices. Using the following notation

$$M_{ab}^{\beta\gamma} = CV^{\beta\gamma,ab}, \tag{A.21}$$

one gets the following properties for $M_{ab}^{\beta\gamma}$

$$M_{12}^{\beta\gamma} + M_{21}^{\beta\gamma} + M_{11}^{\beta\gamma} = CI^{\beta\gamma}, \tag{A.22a}$$
$$[M_{ab}^{\beta\gamma}, M_{cd}^{\beta\gamma}] = 0, \quad \forall\ a, b, c, d, \tag{A.22b}$$
$$M_{12}^{\beta\gamma} M_{21}^{\beta\gamma} = M_{11}^{\beta\gamma\,2} - CI^{\beta\gamma-1} M_{11}^{\beta\gamma}. \tag{A.22c}$$

# B Equivalence of algebraic and conformal solutions.

Using the results on the spectroscopy of the NS star algebra [40] we generalize the technique of Okuyama [27],[31] to the NS case and prove the equivalence of the algebraic and conformal definitions of the NS matter sliver and NS ghost sliver in the minus one picture.

Matrices $F$ and $C\tilde{F}$ have the spectrum

$$\phi(\kappa) = -\frac{1}{\cosh(\frac{\pi\kappa}{2})}, \quad \tilde{\phi}(\kappa) = i\tanh(\frac{\pi\kappa}{2}) \tag{B.1}$$

correspondingly. The corresponding generation function of eigenvectors is given by

$$f_{w^{(\kappa)}}(z) = w_{\frac{1}{2}}^{(\kappa)} \frac{z}{\sqrt{1+z^2}} \exp(-\kappa \arctan(z)). \tag{B.2}$$

The spectrum of the NS matter sliver is given by

$$T(\kappa) = \begin{cases} i\frac{\kappa}{|\kappa|} e^{-\pi|\kappa|/2} & \kappa \neq 0, \\ 0 & \kappa = 0. \end{cases} \tag{B.3}$$

Let us denote as $|z\rangle$ and $|\kappa\rangle$ the infinite dimensional vectors $|z\rangle = (z, z^2, z^3, \dots)^T$ and $|\kappa\rangle = (w_1^{(\kappa)}, w_2^{(\kappa)}, w_3^{(\kappa)}, \dots)^T$. The generating function (B.2) of eigenvectors in these notations is given by

$$f_{w^{(\kappa)}}(z) = w_{\frac{1}{2}}^{(\kappa)} \frac{z}{\sqrt{1+z^2}} \exp(-\kappa \tan^{-1} z) = \sum_{n=1}^{\infty} w_n^{(\kappa)} z^n \equiv \langle z|w^{(\kappa)}\rangle. \tag{B.4}$$

The inner product of two vectors $|w\rangle$ and $|w'\rangle$ is defined as

$$\langle w|w'\rangle \equiv \sum_{n=1}^{\infty} w_n w'_n = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \langle w|e^{i\theta}\rangle\langle e^{-i\theta}|w'\rangle. \tag{B.5}$$

Let us compute the inner product of two eigenvectors

$$\langle\kappa|p\rangle = \int_{-\pi/2}^{3\pi/2} \frac{d\theta}{2\pi} f^*_{w(\kappa)}(e^{i\theta}) f_{w(p)}(e^{i\theta}) = \int_{-\pi/2}^{3\pi/2} \frac{d\theta}{2\pi} \langle\kappa|e^{i\theta}\rangle\langle e^{-i\theta}|p\rangle$$
$$= \int_{-\pi/2}^{3\pi/2} \frac{d\theta}{2\pi} \frac{e^{i\theta}}{\sqrt{1+e^{2i\theta}}} \frac{e^{-i\theta}}{\sqrt{1+e^{-2i\theta}}} e^{-\kappa \tan^{-1} e^{i\theta}} e^{-p \tan^{-1} e^{-i\theta}}. \quad \text{(B.6)}$$

We specify the branch of the function

$$\tan^{-1} z = \frac{1}{2i} \log \frac{1+iz}{1-iz}, \quad \tan^{-1} 0 = 0. \quad \text{(B.7)}$$

We change variables

$$\tan^{-1} e^{i\theta} = \frac{\pi}{4} + ix, \quad \tan^{-1} e^{-i\theta} = \frac{\pi}{4} - ix, \quad \tan\frac{\theta}{2} = \tanh x, \quad \left[-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right] \quad \text{(B.8a)}$$
$$\tan^{-1} e^{i\theta} = -\frac{\pi}{4} - ix, \quad \tan^{-1} e^{-i\theta} = -\frac{\pi}{4} + ix, \quad -\cot\frac{\theta}{2} = \tanh x. \quad \left[\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right] \quad \text{(B.8b)}$$

One finds

$$\langle\kappa|p\rangle = \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-\pi\kappa/4 - i\kappa x} e^{-\pi p/4 + ipx} + \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{\pi\kappa/4 + i\kappa x} e^{\pi p/4 - ipx}$$
$$= e^{-\pi\kappa/2}\delta(\kappa - p) + e^{\pi\kappa/2}\delta(\kappa - p) = 2\cosh\left(\frac{\pi\kappa}{2}\right)\delta(\kappa - p) \equiv \mathcal{N}(\kappa)\delta(\kappa - p). \quad \text{(B.9)}$$

Introducing the normalized eigenvectors $|\hat\kappa\rangle = \mathcal{N}(\kappa)^{-1/2}|\kappa\rangle$ we get the completeness relation

$$1 = \int_{-\infty}^{\infty} d\kappa\, |\hat\kappa\rangle\langle\hat\kappa| = \int_{-\infty}^{\infty} d\kappa\, \mathcal{N}(\kappa)\, |\kappa\rangle\langle\kappa|. \quad \text{(B.10)}$$

It is a good check to consider the following relation

$$\langle z|w\rangle = \sum_{n=1}^{\infty} z^n w^n = \frac{zw}{1-zw} = \int_{-\infty}^{\infty} d\kappa \langle z|\hat\kappa\rangle\langle\hat\kappa|w\rangle =$$
$$= \frac{z}{\sqrt{1+z^2}} \frac{w}{\sqrt{1+w^2}} \int_{-\infty}^{\infty} d\kappa \frac{1}{2\cosh(\frac{\pi\kappa}{2})} e^{-\kappa \tan^{-1} z} e^{-\kappa \tan^{-1} w}$$
$$= \frac{z}{\sqrt{1+z^2}} \frac{w}{\sqrt{1+w^2}} \int_0^{\infty} d\kappa \frac{1}{\cosh(\frac{\pi\kappa}{2})} \cosh(-\kappa(\tan^{-1} z + \tan^{-1} w))$$
$$= \frac{z}{\sqrt{1+z^2}} \frac{w}{\sqrt{1+w^2}} \frac{\sqrt{1+z^2}\sqrt{1+w^2}}{1-zw} = \frac{zw}{1-zw}. \quad \text{(B.11)}$$

Here we have used

$$\int_0^\infty dx\,\frac{\sinh(ax)}{\sinh(bx)}=\frac{\pi}{2b}\tan\frac{\pi a}{2b},\quad (|\Re a|<|\Re b|), \tag{B.12a}$$

$$\int_0^\infty dx\,\frac{\cosh(ax)}{\cosh(bx)}=\frac{\pi}{2b}\frac{1}{\cos\frac{\pi a}{2b}},\quad (|\Re a|<|\Re b|). \tag{B.12b}$$

Further we show the equivalence of the algebraic and conformal sliver. The generating function of the matter NS sliver $S_{rs}$ is given by

$$\begin{aligned}h^{\mu\nu}(z,w)\equiv\langle\Xi|\psi^\mu(w)\psi^\nu(z)|0\rangle&=\langle 0|\exp(-\frac{1}{2}\psi_r S_{rs}\psi_s)\psi^\mu(w)\psi^\nu(z)|0\rangle\\&=-\frac{1}{2}w^{r-1/2}z^{s-1/2}S_{rs}\eta^{\mu\nu}-\frac{1}{2}\eta^{\mu\nu}\frac{1}{w-z}.\end{aligned} \tag{B.13}$$

On the other hand the conformal definition gives

$$h^{\mu\nu}(z,w)\equiv\langle f\circ\psi^\mu(w)f\circ\psi^\nu(z)\rangle=\left(\frac{\partial f(w)}{\partial w}\right)^{1/2}\left(\frac{\partial f(z)}{\partial z}\right)^{1/2}\left(-\frac{1}{2}\right)\frac{\eta^{\mu\nu}}{f(w)-f(z)}. \tag{B.14}$$

Substituting the conformal map $f(z)=\tan^{-1}z$ for the sliver we get the following equation that should hold

$$\langle z|S|w\rangle=\frac{zw}{z-w}+\frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\frac{1}{\tan^{-1}w-\tan^{-1}z}. \tag{B.15}$$

Indeed

$$\begin{aligned}\langle -z|S|w\rangle=\langle z|iCS|w\rangle&=\frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\int_0^\infty d\kappa\, e^{-\frac{\pi\kappa}{2}}\frac{\sinh(\kappa(\tan^{-1}z+\tan^{-1}w))}{\cosh(\frac{\pi\kappa}{2})}\\&=\frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\left[\frac{1}{\sin(\tan^{-1}z+\tan^{-1}w)}-\frac{1}{\tan^{-1}z+\tan^{-1}w}\right]\\&=\frac{zw}{z+w}-\frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\frac{1}{\tan^{-1}z+\tan^{-1}w}.\end{aligned} \tag{B.16}$$

Here we have used

$$\int_0^\infty dx\frac{\sinh(ax)}{e^{bx}+1}=\frac{\pi}{2b}\frac{1}{\sin(\frac{a\pi}{b})}-\frac{1}{2a},\quad p>a,\ p>0. \tag{B.17}$$

The formulae for the fermionic ghosts in the minus one picture are straightforward. The generating function of the fermionic ghost sliver $S^{\beta\gamma}$ in the minus one picture is given by

$$\begin{aligned}h^{\mu\nu}(z,w)\equiv\langle\Xi|\gamma(w)\beta(z)|-1\rangle&=\langle -1|\exp(\gamma_r S^{\beta\gamma}_{rs}\beta_s)\gamma(w)\beta(z)|-1\rangle\\&=w^{r-1/2}z^{s-1/2}S^{\beta\gamma}_{rs}+\frac{1}{w-z}.\end{aligned} \tag{B.18}$$

On the other hand the conformal definition gives

$$h^{\mu\nu}(z,w) \equiv \langle f\circ\gamma(w) f\circ\beta(z)\rangle = \left(\frac{\partial f(w)}{\partial w}\right)^{1/2}\left(\frac{\partial f(z)}{\partial z}\right)^{1/2}\frac{1}{f(w)-f(z)}. \tag{B.19}$$

Substituting the conformal map $f(z)=\tan^{-1}z$ for the sliver we get the following equation that should hold

$$\langle z|S^{\beta\gamma}|w\rangle = \frac{zw}{z-w} + \frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\frac{1}{\tan^{-1}w - \tan^{-1}z}. \tag{B.20}$$

Indeed

$$\begin{aligned}\langle -z|S^{\beta\gamma}|w\rangle = \langle z|iCS^{\beta\gamma}|w\rangle &= \frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\int_0^\infty d\kappa\, e^{-\frac{\pi\kappa}{2}}\frac{\sinh(\kappa(\tan^{-1}z+\tan^{-1}w))}{\cosh(\frac{\pi\kappa}{2})}\\ &= \frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\left[\frac{1}{\sin(\tan^{-1}z+\tan^{-1}w)} - \frac{1}{\tan^{-1}z+\tan^{-1}w}\right]\\ &= \frac{zw}{z+w} - \frac{z}{\sqrt{1+z^2}}\frac{w}{\sqrt{1+w^2}}\frac{1}{\tan^{-1}z+\tan^{-1}w}.\end{aligned} \tag{B.21}$$

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