Time Evolution in Superstring Field Theory on non-BPS brane. I. Rolling Tachyon and Energy-Momentum Conservation

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Abstract

We derive equations of motion for the tachyon field living on an unstable non-BPS D-brane in the level truncated open cubic superstring field theory in the first non-trivial approximation. We construct a special time dependent solution to this equation which describes the rolling tachyon. It starts from the perturbative vacuum and approaches one of stable vacua in infinite time. We investigate conserved energy functional and show that its different parts dominate in different stages of the evolution. We show that the pressure for this solution has its minimum at zero time and goes to minus energy at infinite time.

Contents

1 Introduction

The physical problem which we would like to address in a series of papers is a time dependence of the tension of an unstable non-BPS D-brane in the process of its decay. It is expected that starting from its maximal value, equal to the density of the vacuum energy, the tension of an unstable non-BPS D-brane becomes zero when the tachyon field approaches the true vacuum. In this paper we start with study of classical time dependent solutions describing a motion of the tachyon on an unstable non-BPS D-brane within level truncation scheme in cubic open superstring field theory. In the next paper we will take into account higher level fields including vector fields and study a dynamics of a D-brane tension.

The tachyon potential and various classical solutions in SFT on an unstable D-brane system have been intensively studied in the last two years (see [1, 2, 3, 4, 5], reviews [6, 7, 8] and references therein). In the beginning only time independent solutions representing either the tachyon vacuum or static D-branes of lower dimension have been examined. The process of production and decay of unstable branes can be described as a space-like D-brane [9]. For earlier study of the tachyon dynamics on brane-antibrane system see, for example, [10].

Recently, Sen has proposed to study the classical decay process of an unstable D-brane in the framework of SFT [11, 12, 13, 14]. As compare to the usual local field theory the time dependent equation of SFT containing a tachyon has special features. First, this equation involves infinite number of other fields. Second, the string field theory action contains infinite number of time derivatives, and hence the initial value problem seems to be not well-defined. Nevertheless, Sen [11, 14] has shown that it is possible to construct a family of classical solutions to the string field theory equations of motion characterized by the initial position and velocity of the tachyon field.

There are at least two different ways to construct time dependent solutions within SFT:

- One can study solutions to SFT equation of motion

$$QA + A * A = 0 \tag{1.1}$$

 perturbatively, specifying some initial data. This way has been followed in the background approach to scattering theory in SFT in [15] as well as in Sen's paper [14].

- One can also study a tachyon dynamics within the level truncation scheme. As it has been noted in [16] there are many similarities between the tachyon equation of motion obtained in SFT via the level truncation and the p-adic string equation of motion [17, 18, 19].

The goal of this paper is to study the tachyon dynamics in SSFT on a non-BPS D-brane within the level truncation scheme [20]. We derive the equation of motion for the tachyon in the first nontrivial approximation. We construct a special time dependent solution to this equation which starts from the perturbative vacuum and approaches one of stable vacua. To construct this solution we consider it as a positive time part of a solution that interpolates between two non-perturbative vacua. As in the case of the p-adic string [19] to perform numerical study of the SSFT tachyon equation of motion with boundary conditions on $\pm$ infinities, it is useful to deal with the integral form of the equation of motion. The integral form of the tachyon equation of motion in SSFT is similar to the integral form of the tachyon equation of motion in the p-adic string [19] for $p = 3$ while the integral form of the bosonic SFT is similar to one of the p-adic string for $p = 2$ [16, 28]. In contrast to p-adic case, kernels of the integral equations describing SFT and SSFT tachyons are not positively defined and application of an iteration procedure is more subtle. By our request these simulations have been performed in [29] and we will use these results in the paper. Numerical analysis [29] is based on an iteration procedure of solving nonlinear integral equations. It is interesting to note that SFT and SSFT tachyons equations can be considered as perturbations of the corresponding p-adic string equations of motion. The superstring interpolating tachyon solution can be obtained in a linear approximation around the corresponding p-adic (p=3) solution. This approximation rather well reproduces results of numerical calculations.

In order to understand qualitative properties of solutions we write down the conserved energy functional. The energy E is the sum of three terms, namely the kinetic term E_k, the potential term E_p and the non-local term E_{nl}. There are no difficulties to study numerically the kinetic E_k and potential E_p terms. To investigate numerically contribution of the non-local term E_{nl} we write it in a special form admitting an integral representation.

The pressure is also the sum of three terms

$$P(t) = -E + 2E_k(t) + E_{nlp}(t). \tag{1.2}$$

The non-local term E_{nlp} is just a part of the non-local term E_{nl} contributing to the energy E. We show that for large time $P(\infty) = -E$ while for $t = 0$, under a special approximation for the tachyon solution to the SSFT equations of motion (see Sect. 2.2), $P(0) \approx E$. The energy E is equal to the tachyon potential minimum and according to Sen's conjecture the tension of the non-BPS D-brane compensates this energy and the total energy of a system is equal to zero. This compensation has been checked for cubic SSFT with rather good accuracy [20]. Since the total energy is equal to zero the total pressure also vanishes at large time for our solution. We also show that the pressure for this solution has its minimum at zero time and goes to minus energy at infinite time.

Let us also note that there are other approaches to study rolling tachyons. One of them is related to CFT description of a non-BPS D-brane dynamics. Sen has noted [11, 12, 14] (and refs. therein) that dynamics of a tachyon on an unstable D-brane can be described by an effective Born-Infeld action with an exponentially decreasing potential. Though this action cannot be derived from first principles it has some features desirable in a classical dynamics of open SFT around the tachyon vacuum. Namely, in this effective field theory there are solutions with fixed energy density and asymptotically vanishing pressure. These solutions are also interesting from cosmological point of view, see [21] for details.

Let us note that it would be interesting to see more relations between SFT and the effective Born-Infeld action. An incorporation of vector field could clarify this problem. This hope is supported by the observation that an incorporation of vector fields, apparently, reproduces the Born-Infeld action [22, 23, 24].

The paper is organized as follows. In Section 2 we introduce a general setup, namely the cubic superstring field theory action for a description of an open superstring living on a non-BPS D-brane. Within the level truncation scheme we get from this action tachyon equations of motion. In fact in this paper we restrict ourself to the first non-trivial approximation which includes the tachyon field and one auxiliary field. Under an assumption that even derivatives of the auxiliary field vary slowly we get an approximate version of the tachyon equation of motion. Then we rewrite differential equations in the integral form. In Section 3 we present results of numeric study of the equations obtained in Section 2.

In Section 4 functionals of the energy and pressure are studied. Conservation of energy is calculated explicitly. For the pressure it is shown that it has minimum at zero time and equal to minus energy at infinite time.

2 Equations of Motion for Tachyon Field on Non-BPS D-brane

To describe the open string states living on a single non-BPS D-brane one has to consider GSO$\pm$ states [1]. GSO$-$ states are Grassmann even, while GSO$+$ states are Grassmann odd. The unique (up to rescaling of the fields) gauge invariant cubic action unifying GSO$+$ and GSO$-$ sectors is [20]

$$\begin{aligned} S[A_+,A_-]=\frac{1}{g_o^2}\Bigg[&\frac{1}{2}\langle\langle Y_{-2}|A_+,Q_BA_+\rangle\rangle+\frac{1}{3}\langle\langle Y_{-2}|A_+,A_+,A_+\rangle\rangle\\ &+\frac{1}{2}\langle\langle Y_{-2}|A_-,Q_BA_-\rangle\rangle-\langle\langle Y_{-2}|A_+,A_-,A_-\rangle\rangle\Bigg]. \end{aligned} \tag{2.3}$$

For $A_-=0$ one gets the action [26, 27]. Here the factors before the odd brackets are fixed by the constraint of gauge invariance, that is specified below, and the reality of the string fields $A_\pm$. Variation of this action with respect to A_+ and A_- yields the following equations of motion (see [20] for notations and details)

$$Q_B A_\pm + A_+ \star A_\pm - A_- \star A_\mp = 0. \tag{2.4}$$

The action (2.3) is invariant under the gauge transformations

$$\delta A_\pm = Q_B \Lambda_\pm + [A_\pm, \Lambda_+], +\{A_\mp, \Lambda_-\},$$

where $[\,,]$ $(\{\,,\})$ denotes $\star$-(anti)commutator and $\Lambda_\pm$ are gauge parameters.

To perform actual calculations the level truncation scheme is employed.

Let us denote by $\mathcal{H}_1$ the subset of vertex operators of ghost number 1 and picture 0, created by the matter stress tensor T_B, matter supercurrent T_F and the ghost fields b, c, $\partial\xi$, η and ϕ. We restrict the string fields $\mathcal{A}_+$ and $\mathcal{A}_-$ to be in this subspace $\mathcal{H}_1$.

Next we expand $\mathcal{A}_\pm$ in a basis of L_0 eigenstates, and write the action (2.3) in terms of space-time component fields. The string field is now a series with each term being a vertex operator from $\mathcal{H}_1$ multiplied by a space-time component field. We define the level K of string field's component A_i to be $h+1$, where h is the conformal dimension of the vertex operator multiplied by A_i, i.e. by this convention the tachyon is taken to have level 1/2. To compose the action truncated at level $(K,\,L)$ we select all the quadratic and cubic terms of total level not more than L for the space-time fields of levels not more than K. Since our action is cubic, $2K \le L \le 3K$.

2.1 Action and equations of motion

The level (1/2,1) action contains the tachyon field ϕ and one auxiliary field u

$$S[u,\phi] = \frac{1}{g_o^2 \alpha'^{\frac{(p+1)}{2}}} \int d^{p+1}x \left[u^2(x) - \frac{\alpha'}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) + \frac{1}{4}\phi^2(x) - \frac{1}{3\gamma^2}\tilde{u}(x)\tilde{\phi}(x)\tilde{\phi}(x)\right], \tag{2.5}$$

where the tilde is the action of the operator

$$D_\Box = \exp(-\alpha' \ln\gamma\,\Box), \quad \tilde{\phi} = D_\Box\phi, \quad \text{etc.} \tag{2.6}$$

$\gamma = \frac{4}{3\sqrt{3}}$, $\Box = \partial_\mu\partial^\mu$ and the signature is $(-+++\dots)$. The equations of motion have the form

$$2u(x) - \frac{1}{3\gamma^2} D_\Box\left((D_\Box\phi)^2\right)(x) = 0, \tag{2.7a}$$

$$(\alpha'\Box + 1/2)\,\phi(x) - \frac{2}{3\gamma^2} D_\Box\left((D_\Box\phi)(D_\Box u)\right)(x) = 0. \tag{2.7b}$$

Applying $D_\Box$ to both equations above we can write them as

$$\tilde{u} - \frac{1}{6\gamma^2} D_\Box^2\left((\tilde{\phi})^2\right) = 0, \tag{2.8a}$$

$$(\alpha'\Box + 1/2)\,\tilde{\phi} - \frac{2}{3\gamma^2} D_\Box^2\left(\tilde{u}\tilde{\phi}\right) = 0. \tag{2.8b}$$

Here $D^2_{\Box}\phi \equiv \exp(-2\alpha' \ln\gamma\,\Box)\phi$ and for simplicity we do not write arguments of functions. Performing a rescaling of fields and coordinates

$$\Psi(x) = \frac{\sqrt{2}}{3(\gamma)^2}\tilde{\phi}(\alpha x) \quad \text{and} \quad \Upsilon(x) = \frac{4}{3\gamma^2}\tilde{u}(\alpha x) \tag{2.9}$$

with

$$\alpha^2 = 4a = -4\alpha' \ln\gamma \quad \text{and} \quad q^2 = \frac{\alpha'}{2a} = -\frac{1}{4\ln\gamma}. \tag{2.10}$$

We recast the latter system to the following form

$$\Upsilon(x) - D^2_{\Box}\left(\Psi^2\right)(x) = 0, \tag{2.11a}$$

$$\left(q^2\Box + 1\right)\Psi(x) - D^2_{\Box}\left(\Upsilon\Psi\right)(x) = 0. \tag{2.11b}$$

Hereafter $D_{\Box} = \exp(\frac{1}{8}\Box)$.

2.2 Approximation $u = \tilde{u}$

The approximation $u = \tilde{u}$ is valid when even derivatives of the field u vary slowly. Substituting $u = \tilde{u}$ in action (2.5) we are left with the action which after integration over u and rescaling has the form

$$S[\psi] = \int dx\left[\frac{1}{2}\psi^2(x) - \frac{q^2}{2}\partial_\mu\psi(x)\partial^\mu\psi(x) - \frac{1}{4}\tilde{\psi}^4(x)\right], \tag{2.12}$$

where

$$\tilde{\psi}(x) \equiv \Psi(x) = \exp(\frac{1}{8}\Box)\psi(x). \tag{2.13}$$

Equation of motion for this action is

$$\left(q^2\Box + 1\right)\Psi(x) = D^2_{\Box}\left(\Psi^3\right)(x), \tag{2.14}$$

which can be presented as a system of two equations

$$\Upsilon(x) - \Psi^2(x) = 0, \tag{2.15a}$$

$$\left(q^2\Box + 1\right)\Psi(x) - D^2_{\Box}\left(\Upsilon\Psi\right)(x) = 0. \tag{2.15b}$$

2.3 Integral equations for space homogeneous configurations

For space homogeneous configurations one has

$$\Upsilon(t) - D^2_t\left(\Psi^2\right)(t) = 0, \tag{2.16a}$$

$$\left(-q^2\frac{d^2}{dt^2}+1\right)\Psi(t)-D_t^2\left(\Upsilon\Psi\right)(t)=0. \tag{2.16b}$$

Initial and tilded fields are related as

$$\tilde{\psi}(t)\equiv\Psi(t)=e^{-\frac{1}{8}\frac{d^2}{dt^2}}\psi(t), \tag{2.17a}$$

$$\tilde{\upsilon}(t)\equiv\Upsilon(t)=e^{-\frac{1}{8}\frac{d^2}{dt^2}}\upsilon(t). \tag{2.17b}$$

Note that the operator $D_t^2 = e^{-\frac{1}{4}\frac{d^2}{dt^2}}$ is not well-defined on an arbitrary smooth function. This happens because the operator in the exponent is positively defined. However, we can rewrite the system of equations (2.16) using only the operator $D_t^{-2} = e^{\frac{1}{4}\frac{d^2}{dt^2}}$. The resulting system is

$$D_t^{-2}\Upsilon=\Psi^2, \tag{2.18a}$$

$$\left(-q^2\frac{d^2}{dt^2}+1\right)D_t^{-2}\Psi=\Upsilon\Psi. \tag{2.18b}$$

As in the case of p-adic string [16] one can rewrite this system of differential equations in the integral form

$$C\Upsilon=\Psi^2, \tag{2.19a}$$

$$(-q^2\frac{d^2}{dt^2}+1)C\Psi=\Upsilon\Psi, \tag{2.19b}$$

where

$$\mathcal{C}\Psi\equiv\mathcal{C}_{1/4}\Psi$$

and

$$\mathcal{C}_a\psi(t)=\frac{1}{\sqrt{4\pi a}}\int_{-\infty}^{\infty}e^{-\frac{(t-t')^2}{4a}}\psi(t')dt'. \tag{2.20}$$

The initial fields are related with tilded ones by means of the integral operator C_a

$$\upsilon=\mathcal{C}_{1/8}\Upsilon=\mathcal{C}_{1/8}\tilde{\upsilon}, \tag{2.21a}$$

$$\psi=\mathcal{C}_{1/8}\Psi=\mathcal{C}_{1/8}\tilde{\psi}. \tag{2.21b}$$

In the case $q^2=0$ equations (2.18) take a simple form

$$C\Upsilon=\Psi^2, \tag{2.22a}$$

$$C\Psi=\Upsilon\Psi. \tag{2.22b}$$

2.4 Approximate integral equation

On the space homogeneous configurations instead of (2.14) one gets

$$\left(-q^2\frac{d^2}{dt^2}+1\right)D_t^{-2}\Psi=\Psi^3. \tag{2.23}$$

This equation can be rewritten in the integral form

$$\left(-q^2\frac{d^2}{dt^2}+1\right)\mathcal{C}\Psi=\Psi^3, \tag{2.24}$$

where $\mathcal{C}\Psi\equiv\mathcal{C}_{1/4}\Psi$ and $\mathcal{C}_{1/4}$ is defined in (2.20). If we drop the term with the second derivative we are left with

$$\mathcal{C}\Psi=\Psi^p, \quad p=3. \tag{2.25}$$

This is a p-adic effective field equation for p=3 [17, 19, 16]. For the bosonic string theory one gets equation (2.25) with p=2. As it has been shown in [16] properties of p-adic equation are drastically different for p=2 and p=3. In particular, for p=2 there are no rolling tachyon solutions.

3 Solutions to Integral Equations

3.1 Interpolating solutions

Equation (2.25) has been studied numerically in [19] for p-adic effective field equations for arbitrary p and recently in [14]. More general equation (2.24) as well as a system of equations (2.19) has been studied in [29]. Here we review results of numerical calculations performed in [29]. It was shown that:

- Solution of (2.25) interpolating between two vacua is given by a monotonic function Ψ_0 presented in Fig.1a.
- Solution of (2.24) for $q^2\leq q^2_{cr}=1.38$ is given by a function Ψ interpolating between two vacua (see Fig.1b). This function oscillates near Ψ_0 with exponentially decreasing amplitude.
- Function Ψ solving the system (2.19) for $q^2\leq q^2_{cr}=2.2$ is an odd function with a jump at $t=0$ and $\Psi(\pm\infty)=\mp1$. Υ is even function. In Fig.2a these functions for $q^2\simeq0.96$ are presented.
- The smoothed functions υ and ψ, defined by (2.21), are continuous, see Fig.2b.
- For $q^2\simeq0.96$ function ψ (2.21b) does not differ much from the function obtained from the solution of (2.24), by the same formula (2.21b) $\psi_{app}=C_{1/8}\Psi_{app}$ (see Fig.3).

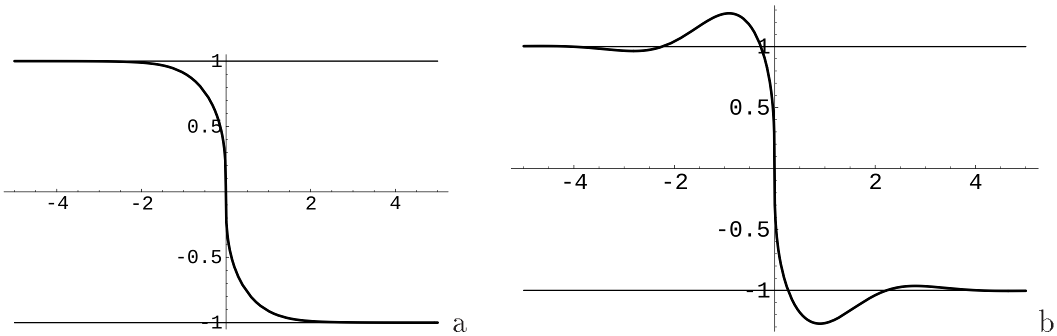


Figure 1: a) $\Psi_0 = \Psi_{app}$ for $q^2 = 0$; b) Ψ_{app} for $q^2 = 0.96$.

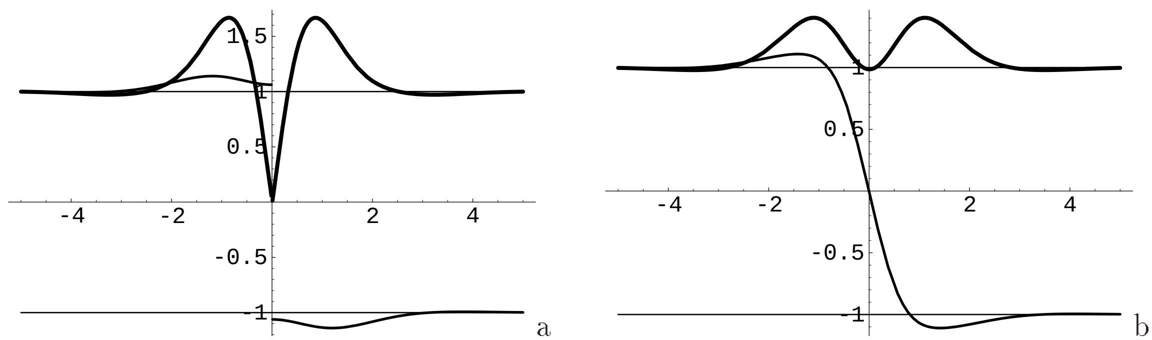


Figure 2: a) Ψ (thin) and Υ (thick) for $q^2 = 0.96$;
b) smoothed functions: ψ (thin) and υ (thick) for $q^2 = 0.96$.

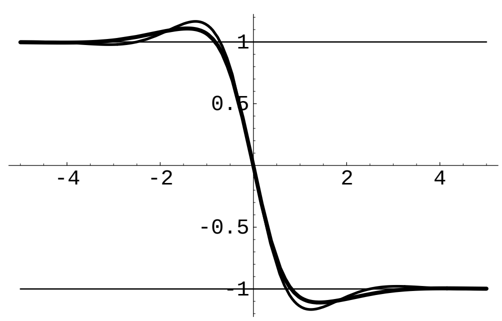


Figure 3: Smoothed functions ψ_{app} (thin) and ψ (thick) of solutions for $q^2 = 0.96$.

3.2 Rolling solutions

The iterative procedure has been formulated so that the function Ψ is an odd one and the function Υ is even. Results of calculations show that the function Υ is smooth while the function Ψ has a discontinuity at the zero time. These results take place for $q \leq q_{cr}$, including $q = 0$. For $q = 0$ one can see that an assumption about the parity properties implies that Ψ in fact has a discontinuity at $t = 0$. Indeed, equations of motion (2.22) can be rewritten as a system of equations on semi-axis:

$$(C_+\Upsilon)(t) = \Psi(t)^2, \tag{3.26a}$$

$$(C_-\Psi)(t) = \Upsilon(t)\Psi(t), \quad 0 \le t < \infty, \tag{3.26b}$$

where C_+ and C_- have kernels K_+ and K_-, respectively,

$$K_+(t,t') = K(t-t') + K(t+t'), \tag{3.27a}$$

$$K_-(t,t') = K(t-t') - K(t+t'), \tag{3.27b}$$

$K(t) = \frac{1}{\sqrt{\pi}}\exp(-t^2)$. These kernels are positive when both $t, t' > 0$. If one assumes that $0 < \Psi(t) < \infty$ for $0 < t < \infty$ than it follows from the equation

$$\Upsilon = \frac{C_-\Psi}{\Psi}, \tag{3.28}$$

that $\Upsilon(t) > 0$ for $0 < t < \infty$. This guaranties that $(C\Upsilon)(t) > 0$ for $0 \le t < \infty$. According to (3.26a) this means that $\Psi^2 > 0$ for $t \ge 0$, i.e. $\Psi^2(0) > 0$. Since Ψ is an odd function it has to have discontinuity at zero time.

This discontinuity disappears when we perform smoothing of this function with the kernel K to obtain the function ψ. In fact this function represents the time evolution of the tachyon field. Shown in thin line, for comparison is ψ_{app} defined as smoothing of the solution to equation (2.24).

We can consider a function $\psi(t)$ starting at zero time from false vacuum and rolling to the true vacuum at large times. For $q = 0$ this is a monotonic function. As it was shown in [29] for $q^2 < q^2_{cr}$ there are small oscillations with decreasing amplitudes around the solutions to equation (2.25). The presence of this oscillations (in fact oscillations with a complex frequency) results from the presence of infinite number of derivatives in a linear equation describing a deviation of $\psi(t)$ from $\psi_{q=0}(t)$.

4 Energy and Pressure

4.1 Conservation of energy

In this section we derive functional expressions for energy for tachyon equations (2.18) and for approximate equation (2.23). Let us consider the system of integral equations (2.19). The conserved energy is a sum of three terms representing kinetic energy E_k, potential energy E_p and non-local terms

$$E(t) = E_k(t) + E_p(t) + E_{nl}(t), \tag{4.29}$$

where

$$E_k(t) = \frac{q^2}{2}(\partial\psi)^2, \tag{4.30}$$

$$E_p(t) = -\frac{1}{4}v^2 - \frac{1}{2}\psi^2 + \frac{1}{2}(\Upsilon\Psi^2), \tag{4.31}$$

$$E_{nl}(t)=\frac{1}{8}\int_0^1 d\rho\left(e^{-\frac{\rho}{8}\partial^2}\Upsilon\Psi\right)\overleftrightarrow{\partial}\left(\partial_t e^{\frac{\rho}{8}\partial^2}\Psi\right)+\frac{1}{16}\int_0^1 d\rho\left(e^{-\frac{\rho}{8}\partial^2}\Psi^2\right)\overleftrightarrow{\partial}\left(\partial e^{\frac{\rho}{8}\partial^2}\Upsilon\right), \quad (4.32)$$

here and below $\partial=\frac{d}{dt}$ and $A\overleftrightarrow{\partial}B=A\partial B-(\partial A)B$. This expression is analogous to an expression for the energy obtained for p-adic equation from calculation of the energy-momentum tensor [16, 28].

One can immediately see that this energy conserves. Indeed,

$$\frac{dE(t)}{dt}=-\frac{1}{2}\upsilon\partial\upsilon+q^2(\partial\psi)(\partial^2\psi)-\psi\partial\psi+\frac{1}{2}(\partial\Upsilon)\Psi^2+\Upsilon\Psi\partial\Psi$$
$$+\frac{1}{8}\int_0^1 d\rho(e^{-\frac{\rho}{8}\partial^2}\Upsilon\Psi)\overleftrightarrow{\partial^2}(\partial e^{\frac{\rho}{8}\partial^2}\Psi)+\frac{1}{16}\int_0^1 d\rho(e^{-\frac{\rho}{8}\partial^2}\Psi^2)\overleftrightarrow{\partial^2}(e^{\frac{\rho}{8}\partial^2}\partial\Upsilon).$$

Using the identity

$$-m\int_0^1 d\rho(e^{-m\rho\partial^2}\varphi)\overleftrightarrow{\partial^2}(e^{-m(1-\rho)\partial^2}\phi)=\varphi\overleftrightarrow{e^{-m\partial^2}}\phi, \quad (4.33)$$

we get

$$\frac{dE(t)}{dt}=-\frac{1}{2}\upsilon\partial\upsilon+q^2(\partial\psi)(\partial^2\psi)-\psi\partial\psi+\frac{1}{2}(\partial\Upsilon)\Psi^2+\Upsilon\Psi\partial\Psi$$
$$-\Upsilon\Psi\overleftrightarrow{e^{-\frac{1}{8}\partial^2}}\partial\psi-\frac{1}{2}\Psi^2\overleftrightarrow{e^{-\frac{1}{8}\partial^2}}\partial\upsilon$$
$$=\frac{1}{2}\partial\upsilon[-\upsilon+e^{-\frac{1}{8}\partial^2}(\Psi^2)]+\partial\psi[(q^2\partial^2-1)\psi+e^{-\frac{1}{8}\partial^2}(\Upsilon\Psi)]=0.$$

On solutions to equation of motion the nonlocal term can be presented as the sum of the following terms

$$E_{nl}(t)=E_{nl1}(t)+E_{nl2}(t)+E_{nl3}(t)+E_{nl4}(t), \quad (4.34)$$

where

$$E_{nl1}(t) = \frac{1}{8}\int_0^1 d\rho\left(e^{\frac{1-\rho}{8}\partial^2}(-q^2\partial^2+1)\psi\right)\left(e^{-\frac{1-\rho}{8}\partial^2}\partial^2\psi\right), \quad (4.35a)$$
$$E_{nl2}(t) = -\frac{1}{8}\int_0^1 d\rho\left(e^{\frac{1-\rho}{8}\partial^2}(-q^2\partial^2+1)\partial\psi\right)\left(e^{-\frac{1-\rho}{8}\partial^2}\partial\psi\right), \quad (4.35b)$$
$$E_{nl3}(t) = \frac{1}{16}\int_0^1 d\rho\left(e^{\frac{1-\rho}{8}\partial^2}\upsilon\right)\left(e^{-\frac{1-\rho}{8}\partial^2}\partial^2\upsilon\right), \quad (4.35c)$$
$$E_{nl4}(t) = -\frac{1}{16}\int_0^1 d\rho\left(e^{\frac{1-\rho}{8}\partial^2}\partial\upsilon\right)\left(e^{-\frac{1-\rho}{8}\partial^2}\partial\upsilon\right). \quad (4.35d)$$

It is interesting to compare the expression (4.34) for the nonlocal part of energy with the nonlocal term corresponding to the energy for equation (2.23). The latter has the form (we use calligraphic letters for quantities defined on solutions to equation (2.23))

$$\mathcal{E}(t)=\mathcal{E}_p+\mathcal{E}_k+\mathcal{E}_{nl}(t), \quad (4.36)$$

where

$$\mathcal{E}_k = \frac{q^2}{2}(\partial\psi)^2, \qquad \mathcal{E}_p = -\frac{1}{2}\psi^2 + \frac{1}{4}\Psi^4, \tag{4.37}$$

and the nonlocal term mentioned above is given by

$$\mathcal{E}_{nl}(t) = \frac{1}{8}\int_0^1 d\rho \left(e^{\frac{-\rho}{8}\partial^2}\Psi^3\right) \overleftrightarrow{\partial} \left(\partial e^{\frac{\rho}{8}\partial^2}\Psi\right). \tag{4.38}$$

Here $\Psi = \Psi_{app}$ and $\psi = \psi_{app}$. This nonlocal term takes the following form on equation of motion

$$\mathcal{E}_{nl}(t) = \mathcal{E}_{nl1}(t) + \mathcal{E}_{nl2}(t) = \frac{1}{8}\int_0^1 d\rho \left(e^{\frac{1-\rho}{8}\partial^2}(-q^2\partial^2+1)\psi\right) \overleftrightarrow{\partial} \left(\partial e^{-\frac{1-\rho}{8}\partial^2}\psi\right). \tag{4.39}$$

Comparing (4.39) to (4.35a) and (4.35b) we see that functional expressions for $\mathcal{E}_{nl1}$ and $\mathcal{E}_{nl2}$ are the same as the ones to E_{nl1} and E_{nl2}.

In Fig.4 we plot functions $\mathcal{E}_p(t) + \mathcal{E}_k(t)$ and $\mathcal{E}_{nl}(t)$. We see that up to inaccuracy in numerical calculations the energy is conserved. To calculate the nonlinear terms numerically we actually use another representation for $\mathcal{E}_{nl}(t)$,

$$\mathcal{E}_{nl}(t) = \frac{1}{8}\int_0^1 d\rho \left(e^{\frac{2-\rho}{8}\partial^2}(-q^2\partial^2+1)\Psi\right) \overleftrightarrow{\partial} \left(\partial e^{\frac{\rho}{8}\partial^2}\Psi\right). \tag{4.40}$$

(4.40) allows us to use the integral form

$$\mathcal{E}_{nl}(t) = \frac{1}{8}\int_\varepsilon^1 d\rho (K_1\Psi)(K_2\Psi) - \frac{1}{8}\int_\varepsilon^1 d\rho (K_3\Psi)(K_4\Psi), \tag{4.41}$$

where $K_1 = (-q^2\partial^2+1)C_{\frac{2-\rho}{8}}$, $K_2 = \partial^2 C_{\frac{\rho}{8}}$, $K_3 = (-q^2\partial^3+\partial)C_{\frac{2-\rho}{8}}$, $K_4 = \partial C_{\frac{\rho}{8}}$. In actual calculations we take $\varepsilon \simeq 0.02$.

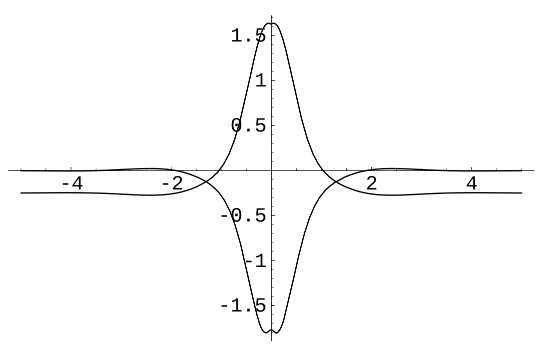


Figure 4: Testing the constancy of the energy for the interpolating solution to equation (2.24). This figure shows $\mathcal{E}_p(t) + \mathcal{E}_k(t)$ and $\mathcal{E}_{nl}(t)$, calculated from (4.37) and (4.41), respectively. Their sum is equal to zero up to numerical errors.

4.2 Pressure

The pressure is defined in terms of the energy-momentum tensor $P(t)_i = -T^i_i$ (no summation). Since we consider homogeneous configurations we will omit the vector index i. The pressure has the form

$$P(t) = -E + 2E_k(t) + 2E_{nl2}(t) + 2E_{nl4}(t), \tag{4.42}$$

or explicitly

$$\begin{aligned} P(t) = -E + q^2(\partial\psi)^2 - \frac{1}{4}\int_0^1 d\rho (e^{\frac{1-\rho}{8}\partial^2}(-q^2\partial^2+1)\partial\psi)(e^{-\frac{1-\rho}{8}\partial^2}\partial\psi) \\ -\frac{1}{8}\int_0^1 d\rho (e^{\frac{1-\rho}{8}\partial^2}\partial\upsilon)(e^{-\frac{1-\rho}{8}\partial^2}\partial\upsilon). \end{aligned} \tag{4.43}$$

The analogous formula takes place for bosonic SFT [16, 28]. For equation (2.24) the pressure $\mathcal{P}$ is

$$\mathcal{P}(t) = -E + 2\mathcal{E}_k(t) + 2\mathcal{E}_{nl2}(t), \tag{4.44}$$

i.e. the functional expression for this pressure is given by

$$\mathcal{P}(t) = -E + q^2(\partial\psi)^2 - \frac{1}{4}\int_0^1 d\rho (e^{\frac{1-\rho}{8}\partial^2}(-q^2\partial^2+1)\partial_t\psi)(e^{-\frac{1-\rho}{8}\partial^2}\partial_t\psi). \tag{4.45}$$

For a large time the pressure in both cases is equal to $-E$.

One can see that the pressure (4.45) at the zero time is equal to the total energy E, i.e. $\mathcal{P}(0) = E$. This can be shown by taking into account the explicit relation between the energy and the pressure (4.44) and the fact that for our solution $\mathcal{E}_{nl1}(0) = 0$. Indeed,

$$\mathcal{P}(0) = -E + q^2(\partial\psi)^2(0) + \mathcal{E}_{nl2}(0) = -E + q^2(\partial\psi)^2(0) + 2E - q^2(\partial\psi)^2(0) = E.$$

For the case of system of equations (2.19) the pressure at $t = 0$ is given by

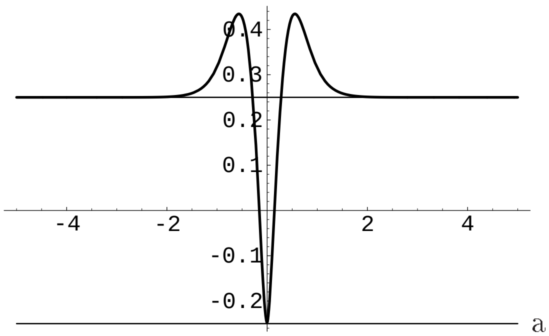
$$P(0) = -E + q^2(\partial\psi)^2(0) - \frac{1}{4}\int_0^1 d\rho \left(e^{\frac{1-\rho}{8}\partial^2}(-q^2\partial^2+1)\partial\psi\right)\left(e^{-\frac{1-\rho}{8}\partial^2}\partial\psi\right). \tag{4.46}$$

One sees that this functional expression coincides with the expression (4.45) that gives the pressure for equation (2.24). In Fig.5 we show the pressure $\mathcal{P}(t)$ for equation (2.24) as a function of time. To obtain this picture we use

$$\mathcal{P}(t) = -E + q^2(\partial\psi)^2(0) - \frac{1}{4}\int_\varepsilon^1 d\rho (K_3\Psi)(K_4\Psi). \tag{4.47}$$

The minimum of the pressure is at t=0. Note that for large times the value of $\mathcal{P}$ is positive and equal to $-E = 1/4$.

In the above consideration we take into account only the energy of tachyon field. Following the Sen conjecture one has to add to the action an extra term representing the D-brane tension. This term shifts on a constant the tachyon potential making it equal to zero at its minimum. This constant also shift the pressure [28] and we get that for our solutions the pressure goes to zero for large times.



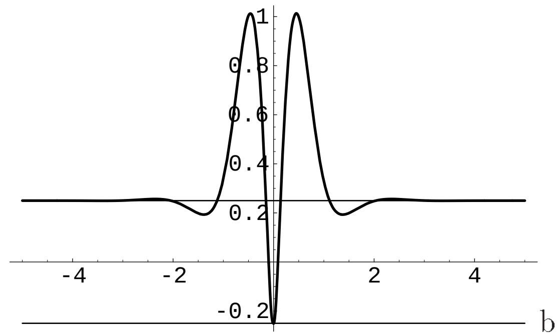


Figure 5: Calculating the pressure on the solutions Fig.1 to equation (2.24); a) for $q = 0$, b) for $q^2 = 0.96$. This figure shows $\mathcal{P}(t)$ has minimum at $t = 0$ and goes to 1/4 at the $\pm\infty$

5 Conclusion

In this paper we have studied the SSFT tachyon classical time dependent equations of motion describing non-stability of non-BPS D-branes. We have found the special time dependent solution to this equation that describes the rolling tachyon. It starts from the perturbative vacuum and approaches one of stable vacua in infinite time. We have argued that the pressure on this solution tends to zero for large times. These results have been obtained in the first non-trivial level within the level truncation method. In the next paper we will take into account higher level fields including vector fields and study a dynamics of a D-brane tension.

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