

**Logische Complexiteit, proeftentamen**  
(open boek) 8.10.2004

1. Define the Satisfiability problem SAT and formulate Cook's theorem.
2. Prove using the pumping lemma that the language  $L = \{0^n 10^n : n \geq 0\}$  is not regular.
3. Is  $\overline{\text{HALT}} \leq_p \text{HALT}$ ?
4. Find  $d = \text{gcd}(126, 330)$  following Euclid algorithm. Find integers  $n$  and  $m$  such that  $126n + 330m = d$ . (Using a calculator is not allowed in this problem.)
5. Show that the problem of testing whether a number is composite (not prime) is in the class NP.
6. Let ISO-CLIQUE be the set of all (codes of) triples  $\langle G, m, k \rangle$  such that  $G$  is a graph,  $G$  has a clique of size  $k$  and  $G$  has (at least)  $m$  isolated points ( $k, m \leq |G|$ ). Show that ISO-CLIQUE is NP-complete. Hint: construct a  $p$ -reduction from CLIQUE to ISO-CLIQUE.
7. Is CLIQUE  $\leq_p$  TQBF? (Motivate your answer.)  
Show: If TQBF  $\leq_p$  CLIQUE, then PSPACE = NP.