Uitwerking Tussentoets 2003

Opgave 3. Let $f : \mathbf{N} \to \mathbf{N}$ be a total computable function and let $X \subseteq \mathbf{N}$ be an r.e. set. Show that $Y = \{x \in \mathbf{N} : f(x) \in X\}$ is r.e.

Uitwerking. We show that there is a partial computable function g such that $dom(g) = Y$. Since X is r.e., there is a partial computable function h such that $dom(h) = X$. We define: $g(x) = h(f(x))$, in other words, g is the composition of h and f. On input x one can compute $y = f(x)$ and then compute $h(y)$. Hence, g is a computable function. Since f is everywhere defined (total), $g(x)$ is defined if and only if $h(f(x))$ is defined, that is, $f(x) \in dom(f) = X$.

Opgave 4. Is the class of decidable languages closed under $*$?

Uitwerking. Yes, it is. Suppose A is a decidable language (decided by a TM M). A^* is defined by

$$A^* = \{w \in \Sigma^* : \exists n \in \mathbf{N}, \exists x_1 \in A, \ldots, \exists x_n \in A \; w = x_1 x_2 \ldots x_n\}.$$

Thus, to check whether a given word w belongs to A^* one has to consider all possible splittings of w into subwords $w = x_1 x_2 \ldots x_n$. (There are only finitely many possible splittings of a given word w.) For every such splitting, one can test whether all words x_i are from A by running the machine M on each of them (n times). If we find any such splitting, we conclude that $w \in A^*$. If no such splitting is found, $w \notin A^*$.