

Logics for AI, Examination problems 2003

1. Give a definition of bisimulation and zig-zag equivalence between Kripke models.
2. Determine, if the following formulas are provable in **S5** (give a derivation, if yes, and a countermodel, if no):
 - (a) $\diamond\Box\Box p \rightarrow \Box p$;
 - (b) $\Box p \rightarrow \neg\Box\Box q$.
3. Show that the following set of formulas is **K4**-consistent:
 $\{\diamond(q \wedge \neg p \wedge \Box(\neg p \wedge q)) \wedge \diamond r \wedge \diamond\neg r, \diamond(p \wedge \neg q \wedge r)\}$
Draw an appropriate Kripke model.
4. Show that there is a node x in the canonical model \mathcal{W}^c for **K** such that $\mathcal{W}^c, x \models \Box\varphi \rightarrow \varphi$, for any formula φ ;
5. The three card problem (hexagon). There are three cards 0,1,2 and three players a, b, c . Each takes one card and keeps it secret.
 - (a) Draw the Kripke model representing the knowledge situation using the variables of the form q_a, q_b, q_c with $q = 0, 1, 2$ (1_a means a has the card 1, etc.).
 - (b) Assume that it is publicly announced by the referee who sees all the players cards: “If a has 0, then b has 1”. Write down the update formula. Draw the updated model.
 - (c) It is assumed that the players are **S5** reasoners and can make only true statements. Suppose a says: “If I have 0, then b has 1”. Write down the update formula. Draw the new model representing the updated knowledge.