Mathematics for Neural Networks
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1. (a) Find a general solution of the system of linear equations

$$\begin{cases} x_1 + x_2 - x_3 + x_4 = 2 \\ x_1 - x_2 + x_3 - x_4 = 0 \\ 3x_1 + x_2 - x_3 + x_4 = 4 \\ 3x_1 - x_2 + x_3 - x_4 = 2 \end{cases}$$

   (b) Find $rk(A)$ where A is the matrix of the system.
   (c) Find a basis in the space of solutions of the homogeneous system $A\vec{x} = 0$.

2. Let A be the matrix

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & -2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

   Find A^{-1}.

3. Let $\mathcal{A} : \mathbf{R}^n \to \mathbf{R}^m$ be a linear transformation and $L \subseteq \mathbf{R}^m$ a subspace of $\mathbf{R}^m$. Prove that $M = \{\vec{x} \in \mathbf{R}^n : \mathcal{A}(\vec{x}) \in L\}$ is a vector space.

4. Consider the vectors $\vec{u} = (1, 3, 0, 2)$ and $\vec{v} = (0, -1, 1, 0)$.

   (a) Are these vectors linearly independent?

   (b) Does the vector $\vec{z} = (2, 9, -3, 4)$ belong to the linear span $L = \langle \vec{u}, \vec{v} \rangle$ of $\vec{u}$ and $\vec{v}$? If so, find the coordinates of $\vec{z}$ in the basis $\vec{u}, \vec{v}$ of L.

5. Let $\mathcal{A} : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation of rotation w.r.t. the origin to the angle $60°$ (counterclockwise). Find the matrix of the transformation $\mathcal{A}$ in the standard basis. Find the matrix of the inverse transformation.