Giant magnons in TsT-transformed $AdS_5 \times S^5$

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Based on joint work arXiv:0805.1070 with Sergey Frolov

AdS/CFT conjecture

$\mathcal{N} = 4$ Super-Yang-Mills theory

Maldacena '97

$\Updownarrow$

IIB strings on $AdS_5 \times S^5$

$$SU(2,2) \times SU(4) \subset PSU(2,2|4)$$

Conformal group in 4 dimensions	R-symmetry of $\mathcal{N} = 4$
Isometry group of AdS_5	Isometry group of S^5

Scaling dimension in gauge theory $\longleftrightarrow$ Energy of string

The deformed space $AdS_5 \times S^5_\gamma$

Lunin, Maldacena '05

$$\mathcal{L}_{S^5_\gamma} = \gamma^{\alpha\beta}\left(\partial_\alpha\rho_i\partial_\beta\rho_i + G\,\rho_i^2\partial_\alpha\varphi_i\partial_\beta\varphi_i + G\,\rho_1^2\rho_2^2\rho_3^2\left(\hat\gamma_i\partial_\alpha\varphi_i\right)\left(\hat\gamma_j\partial_\beta\varphi_j\right)\right)$$
$$-2\,G\,\epsilon^{\alpha\beta}\left(\hat\gamma_3\rho_1^2\rho_2^2\partial_\alpha\varphi_1\partial_\beta\varphi_2 + \hat\gamma_1\rho_2^2\rho_3^2\partial_\alpha\varphi_2\partial_\beta\varphi_3 + \hat\gamma_2\rho_3^2\rho_1^2\partial_\alpha\varphi_3\partial_\beta\varphi_1\right)$$

$$G^{-1} = 1 + \hat\gamma_3^2\rho_1^2\rho_2^2 + \hat\gamma_1^2\rho_2^2\rho_3^2 + \hat\gamma_2^2\rho_1^2\rho_3^2\,, \qquad \sum_{i=1}^{3}\rho_i^2 = 1$$

The TsT transformation

3-torus T^3 in S^5
formed by (ϕ_1, ϕ_2, ϕ_3)

$$\sum_{i=1}^{6} x_i^2 = 1 \qquad x_1 + i\,x_2 = \rho_1 e^{i\phi_1} \quad x_3 + i\,x_4 = \rho_2 e^{i\phi_2} \quad x_5 + i\,x_6 = \rho_3 e^{i\phi_3}$$

Three 2-tori $(\phi_1, \phi_2),\ (\phi_1, \phi_3),\ (\phi_2, \phi_3) \subset T^3$

TsT ↓ **TsT** ↓ **TsT** ↓

S^5_γ

Frolov '05

The dual superconformal $\mathcal{N}=1$ theory superpotential:

$$W = \mathrm{tr}(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2), \qquad \gamma = \frac{\widehat{\gamma}}{2\pi g} = \frac{\widehat{\gamma}}{\sqrt{\lambda}}$$

Leigh, Strassler '95

Action for strings propagating in S^3 in the conformal gauge

$$S = -\frac{g}{2}\int_{-r}^{r} d\sigma d\tau \left(\frac{\partial_\alpha\chi\partial^\alpha\chi}{4\chi(1-\chi)} + (1-\chi)\partial_\alpha\phi_1\partial^\alpha\phi_1 + \chi\partial_\alpha\phi_2\partial^\alpha\phi_2\right).$$

$$\chi = \cos^2\theta$$

Twisted boundary conditions:

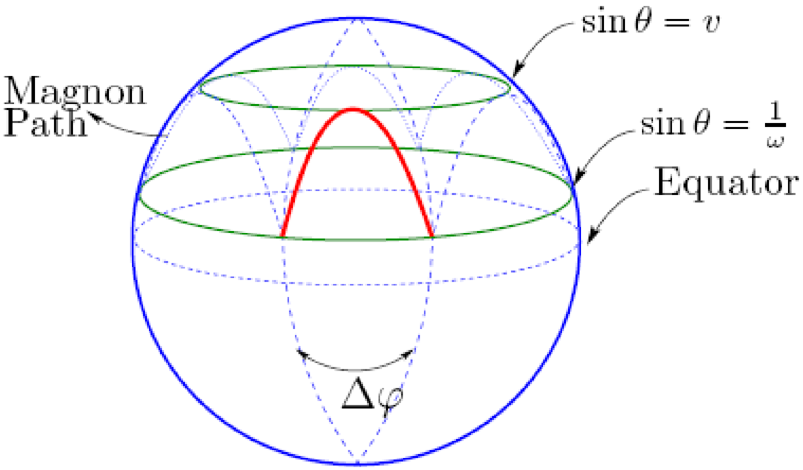
$$\chi(r) = \chi(-r)$$

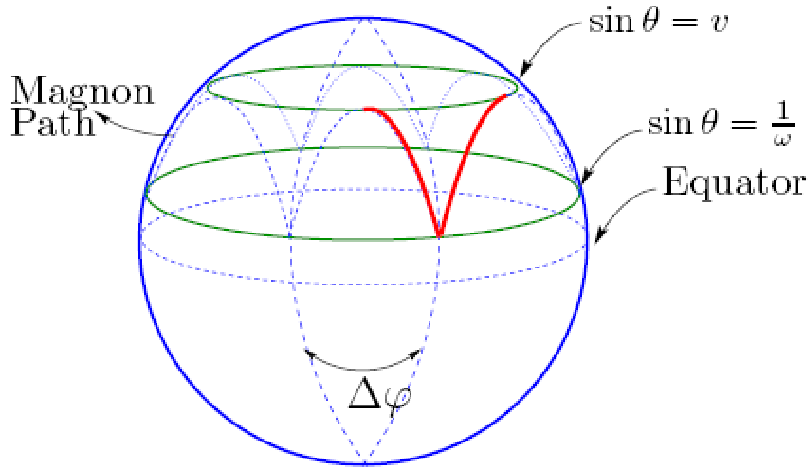
$$\Delta\phi_1 = \phi_1(r) - \phi_1(-r) = p$$

$$\Delta\phi_2 = \phi_2(r) - \phi_2(-r) = 2\pi(n_2 - \gamma J)$$

Giant magnons

Hofman, Maldacena '06





$$O_p \sim \sum_l e^{ilp} \left(...ZZZWZZZ...\right)$$

Dispersion relation on the infinite world-sheet

$$E - J = \sqrt{1 + 4g^2 \sin^2 \frac{p}{2}}$$

$\min\,(E-J)$ at fixed $\lambda\,,\ p$

The giant magnon ansatz

$$\phi_1(\sigma,\tau) = \omega\tau + \frac{p}{2r}(\sigma - v\tau) + \phi(\sigma - v\tau)\,,$$

$$\phi_2(\sigma,\tau) = \nu\tau + \frac{\delta}{2r}(\sigma - v\tau) + \alpha(\sigma - v\tau)\,, \qquad \mathcal{J}_1 \equiv \mathcal{J}\,,\ \mathcal{J}_2 = 0$$

$$\chi = \chi(\sigma - v\tau)$$

Independent parameters $p,\ \delta,\ \mathcal{J}$

$$v(\mathcal{J}) = \cos\frac{p}{2} - \frac{4}{e^2}\sin\frac{p}{2}\,\cos\frac{p}{2}\,\cos\Phi\,\left(\sin\frac{p}{2} + \mathcal{J}\right)\,e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots$$

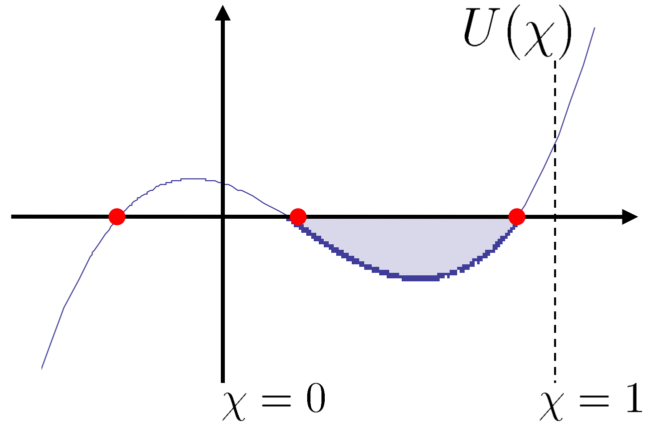
$$\omega(\mathcal{J}) = 1 + \frac{8}{e^2}\sin^2\frac{p}{2}\,\cos\Phi\,e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots\,,$$

$$\nu(\mathcal{J}) = \frac{4}{e^2}\cos\frac{p}{2}\,\sin\Phi\,\left(2\sin\frac{p}{2} + \mathcal{J}\right)\,e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots$$

Equations of motion and Virasoro conditions

$$\phi' = -\left(\frac{v\omega}{1-v^2} + \frac{p}{2r}\right) - \frac{vA_1}{1-v^2}\frac{1}{1-\chi}$$

$$\alpha' = -\left(\frac{v\nu}{1-v^2} + \frac{\delta}{2r}\right) - \frac{vA_2}{1-v^2}\frac{1}{\chi}$$



$$\frac{(1-v^2)^2}{4}\chi'^2 = \kappa_0 + \kappa_1\chi + \kappa_2\chi^2 + \kappa_3\chi^3 \equiv -U(\chi)$$

↑

Torus —— Infinite $\mathcal{J}$ limit ——→ Cylinder

We label the roots of r.h.s. as $\chi_{\text{neg}}, \chi_{\text{min}}, \chi_{\text{max}}$

The resulting system of equations:

$$\text{Periodicity of } \phi : \quad \frac{rv\omega}{1-v^2} + \frac{p}{2} = -\frac{v\,A_1}{1-v^2} \int\limits_{\chi_{\min}}^{\chi_{\max}} \frac{d\chi}{(1-\chi)|\chi'|};$$

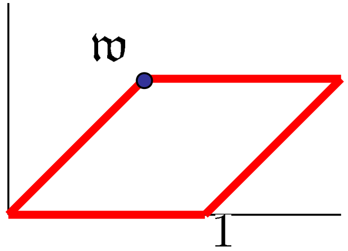
$$\text{Periodicity of } \alpha : \quad \frac{rv\nu}{1-v^2} + \pi\delta = -\frac{v\,A_2}{1-v^2} \int\limits_{\chi_{\min}}^{\chi_{\max}} \frac{d\chi}{\chi|\chi'|};$$

$$\text{Charge } \mathcal{J} \equiv J_1 g : \quad \mathcal{J} = \frac{2}{1-v^2} \left(rA_1 v^2 + \omega \int\limits_{\chi_{\min}}^{\chi_{\max}} d\chi \, \frac{(1-\chi)}{|\chi'|}) \right) ;$$

$$\text{Charge } \mathcal{J}_2 \equiv J_2 g = 0 : \quad 0 = rv^2 A_2 + \nu \int\limits_{\chi_{\min}}^{\chi_{\max}} d\chi \, \frac{\chi}{|\chi'|},$$

$$\text{Length of string:} \quad \int_{-r}^{0} d = r = \int\limits_{\chi_{\min}}^{\chi_{\max}} \frac{d\chi}{|\chi'|} \ .$$

Expansions in the decompactification limit



Modular parameter of the torus $\mathfrak{w} = i\,K(1-\epsilon)$

$$\epsilon(J) = \frac{16}{e^2} e^{-\frac{\mathcal{J}}{\sin\frac{p}{2}}} \left[1 - \frac{8}{e^2} e^{-\frac{\mathcal{J}}{\sin\frac{p}{2}}} \left(1 - \mathcal{J}\,\frac{2 - 3\sin^2\frac{p}{2}}{2\sin\frac{p}{2}} \cos(\Phi) - \frac{1}{2}\mathcal{J}^2\,\cot^2\frac{p}{2}\cos\Phi\right) + \ldots\right].$$

$$\chi_{\rm neg}(\mathcal{J}) = -\frac{16}{e^2}\sin^2\frac{p}{2}\,\sin^2\frac{\Phi}{2}\,e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots$$

$$\chi_{\rm min}(\mathcal{J}) = \frac{16}{e^2}\sin^2\frac{p}{2}\,\cos^2\frac{\Phi}{2}\,e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots$$

$$\chi_{\rm max}(\mathcal{J}) = \sin^2\frac{p}{2} + \frac{8}{e^2}\sin\frac{p}{2}\,\cos^2\frac{p}{2}\,\cos\Phi\,\left(3\,\sin\frac{p}{2} + \mathcal{J}\right)e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots$$

The dispersion relation

$$E - J = 2g\sin\left(\frac{p}{2}\right)\left(1 - \frac{4}{e^2}\sin^2\left(\frac{p}{2}\right)\boxed{\cos\Phi}\, e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots\right)$$

$$\Phi = \frac{2\pi\left(n_2 - \gamma J\right)}{2^{3/2}\cos^3\left(\frac{p}{4}\right)}$$

- Relation not modified in the infinite $\mathcal{J}$ limit

  contrary to **Chu, Georgiou, Khoze '06**

  No shift $p - 2\pi\gamma$

  Relation between world-sheet and spin-chain momenta $p = p_{ws} + 2\pi\gamma(J_2 - J_3)$

- $\Phi = 0$ – coincides with **Arutyunov, Frolov, Zamaklar '06**

$$E - J = 2g\sin\left(\frac{p}{2}\right)\left(1 - \frac{4}{e^2}\,\sin^2\left(\frac{p}{2}\right)\boxed{\cos\Phi}\,e^{-\frac{\mathcal{J}}{\sin(p/2)}} + \ldots\right)$$

$$\Phi = \frac{2\pi(n_2 - \gamma J)}{2^{3/2}\cos^3\left(\frac{p}{4}\right)}$$

- Energy of the solution in deformed theory is higher

- Solution exists if and only if

$-\pi < \Phi \leq \pi$ **all values of** p → $-\frac{1}{2} < n_2 - \gamma J \leq \frac{1}{2}$ → Unique

Summary and prospects

- Obtained the dispersion relation for giant magnons in γ -deformed theory
- A challenge for Luescher's approach to finite size corrections

  (How can we obtain the $\cos\Phi$ in the formula?)

- Solutions with several spins
- Loop corrections $\left(\frac{1}{g}\right)$
- Nonsupersymmetric γ_i background