Introduction to the AdS/CFT correspondence

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The main concept

- Quantum field theory in $\mathbf{R}^{D}$ can be described by gravity in AdS_{D+1} Maldacena, 1997
- Especially strong quantitative evidence in the conformal case, however we will also describe the non-conformal setup
- The extra dimension has the interpretation of RG-scale in QFT

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The AdS space

- A non-compact manifold with constant negative curvature and Lorentzian signature
- Can be viewed as a quotient space (coset) $AdS_{D+1} = \frac{SO(2,D)}{SO(1,D)}$
- For the moment let us set $D = 2$ and consider AdS_3
- The simplest model: a hyperboloid

$$-Y_{-1}^2 - Y_0^2 + Y_1^2 + Y_2^2 = -R^2$$ inside $\mathbb{R}^{2,2}$:

$$ds^2 = -dY_{-1}^2 - dY_0^2 + dY_1^2 + dY_2^2$$

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The AdS space. 2.

- The global coordinates

  $$Y_{-1} = \cosh(r)\,\cos(t),\; Y_0 = \cosh(r)\,\sin(t),$$

  $$Y_1 = \sinh(r)\,\cos(\phi),\; Y_2 = \sinh(r)\,\sin(\phi)$$

  $$ds^2 = -\cosh(r)^2\,dt^2 + dr^2 + \sinh(r)^2 d\phi^2,$$

  $$r \in [0, \infty),\; t \in (-\infty, \infty),\; \phi \in [0, 2\pi)$$

- A conformal rescaling $ds^2 = \cosh(r)^2\,d\tilde{s}^2$ and change of coordinates $w = 2\arctan(e^r) - \frac{\pi}{2}$ produces a metric $$d\tilde{s}^2 = -dt^2 + dw^2 + \sin(w)^2\,d\phi^2$$ where $w \in [0, \frac{\pi}{2}] \Rightarrow$ Disc

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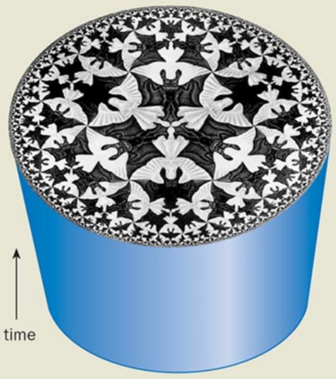
- Hence we arrive at the Penrose diagram of *AdS* — a cylinder:



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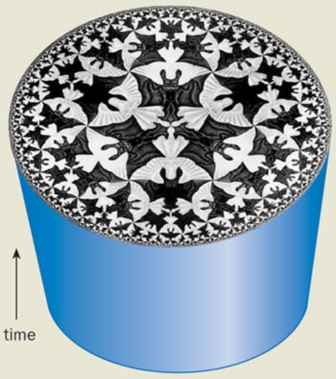

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- *AdS* has a boundary, i.e. a hypersurface reflecting light rays: $r = \infty$
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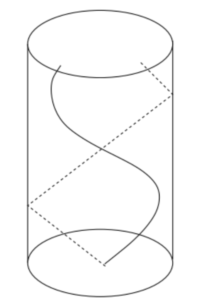

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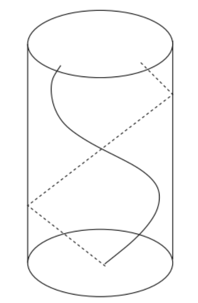

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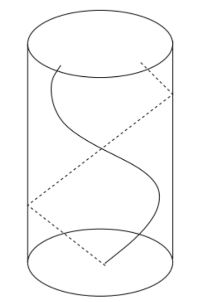

Partition functions

- The correspondence can be made more precise
  Gubser, Klebanov, Polyakov; Witten, 1998
- To every field φ in the bulk of AdS there corresponds a local operator $O(x)$ on the boundary
- For example, $g_{\mu\nu} \Leftrightarrow T_{\mu\nu}, \quad A_\mu \Leftrightarrow J_\mu$
- Moreover, $\left\langle e^{i\int d^4x\, \phi_0(x)\, O(x)} \right\rangle = \mathcal{Z}_{bulk}(\phi_0)$
  — the partition function of the bulk theory with the condition $\phi \to \phi_0$ at the boundary
- $i \int d^4x\, A^\mu_{\rm B}(x)\, J_\mu(x)$ is gauge-invariant!

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- Imagine a scalar field φ in the bulk of AdS with the action $\mathcal{S} = \int d^5y\sqrt{-g}\left((\nabla\varphi)^2 + m^2\varphi^2\right)$ obeying a wave equation $-\triangle\varphi + m^2\varphi = 0,$ where $\triangle = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\,\cdot)$ is the Laplacian.
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$$ds^2 = \frac{-dt^2 + d\vec{x}^2_{d-1} + dz^2}{z^2}$$

. The boundary is at $z = 0$. The equation takes the form (after the ansatz $\varphi \sim e^{i\,k\cdot x} f(z)$)

$$f'' + \frac{1-d}{z} f' - (k^2 + \frac{(mR)^2}{z^2})\, f = 0$$

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  Therefore the solution is a power function $f \sim z^{\Delta}$

- Plugging the ansatz into the equation and solving for Δ, one obtains

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + (mR)^2}.$$

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- Therefore we impose the condition
$$\varphi(x,z)\big|_{z\to 0} \to z^{\Delta_-}\,\varphi_0(x)$$
- To solve the equation with this b.c. one needs to find a function $K(\vec{x},\vec{y},z)$ — the boundary-to-boundary propagator — with the property $K(\vec{x},\vec{y},z)\big|_{z\to 0} \to z^{\Delta_-}\,\delta(\vec{x}-\vec{y})$. Then the desired solution may be written as
$$\varphi(\vec{x},z) = \int d^4y\, K(\vec{x},\vec{y},z)\,\varphi_0(y)$$
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$$K = \Big(\frac{z}{z^2+(\vec{x}-\vec{y})^2}\Big)^{\Delta_+}$$

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Partition functions. 6.

- To compute the path integral for the scalar field φ with the prescribed b.c. we calculate the value of the action S on the solution:

$$\mathcal{S} \sim \int d^D x\, \partial_\mu(\sqrt{-g}\, g^{\mu\nu} \phi \partial_\nu \phi) =$$

$$= -\int d^D x\, \frac{1}{\epsilon^{D-1}} \varphi\, \frac{\partial \varphi}{\partial z}|_{z=\epsilon}$$

- For $z \to 0$ we have the asymptotics $\varphi \to z^{\Delta_-} \varphi_0(x) + z^{\Delta_+} \int d^4 y\, \frac{\varphi_0(y)}{|x-y|^{2\Delta_+}}$, so

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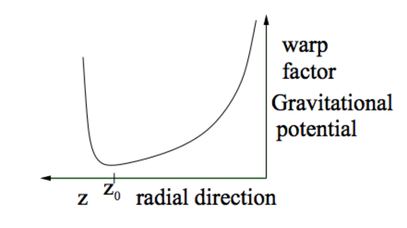
A confining potential

- Let us assume that the AdS warp factor $\frac{1}{z^2}$ is cut-off at some finite value z_0 of the coordinate z



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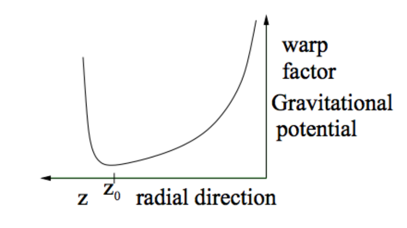

A confining potential. 2.

- The solution of the equation

$$f'' + \frac{1-d}{z} f' - \left(k^2 + \frac{(mR)^2}{z^2}\right) f = 0$$

may be written as

$$f(k,z) = e^{i\,k\cdot x}\, z^{d/2}\, K_\nu(|k|\,z), \quad \nu = \frac{\Delta_+ - \Delta_-}{2}$$

- The boundary condition $f(k, z_0) = 0$ leads to the quantization of $k_n^2 \equiv \lambda_n$ — the spectrum of masses of the mesons

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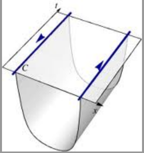
- An important observable in a QFT is the Wilson loop $\langle \mathbf{tr}\left(\mathbf{P}\exp\left[i\int_C dt\, A_\mu \dot{x}^\mu\right]\right)\rangle$, which depends on the contour C in spacetime
- AdS/CFT provides a method for calculating it Maldacena, 1998
- At large 't Hooft coupling λ one should find a minimal area surface ending on the Wilson loop at the boundary



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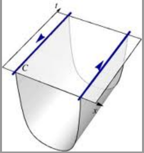

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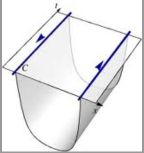

The Wilson loop

- An important observable in a QFT is the Wilson loop $\left\langle \mathbf{tr}\left(\mathbf{P}\,\mathbf{exp}\left[i\int_C dt\,A_\mu \dot{x}^\mu\right]\right)\right\rangle$, which depends on the contour C in spacetime
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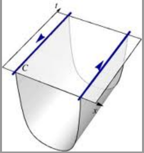

The quark-antiquark potential

- Let us calculate the Wilson loop depicted above at strong coupling
- The area is $\mathcal{A} \sim \int d\sigma\, d\tau \sqrt{\det h}$ with h the induced metric: $h_{ab} = \frac{\partial X^M}{\partial \sigma^a} \frac{\partial X_M}{\partial \sigma^b}$
- AdS_3 with coordinates (X, T, Z). We set $X = \sigma, T = \tau, Z = Z(X)$, therefore

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$$X = \pm \int_{Z}^{z_m} \frac{\left(\frac{z}{z_m}\right)^2 dz}{\sqrt{1-\left(\frac{z}{z_m}\right)^4}}$$

- The regularized area is

$$A \sim 2T \int_{\epsilon}^{z_m} \frac{dz}{z^2 \sqrt{1-\left(\frac{z}{z_m}\right)^4}} \sim T\left[\frac{1}{\epsilon} - \frac{\sqrt{2\pi}\,\pi}{\Gamma(\frac{1}{4})^2 z_m} + \cdots\right]$$

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- At a large separation L the arcs will be negligible, and the area will be proportional to the area of the rectangle, i.e. $\langle W \rangle \sim e^{-\frac{TL}{z_0^2}}$
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- $\mathcal{N} = 4$ SYM — the maximally supersymmetric conformal QFT in $D = 4$
- A vector multiplet + 3 chiral multiplets (in $\mathcal{N} = 1$ superspace)
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- Properties of the AdS space
- Semiclassical calculation of the scalar field partition function
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