

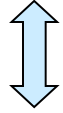
Semiclassical quantization of spinning strings in $AdS_4 \times CP^3$

DMITRI BYKOV

Trinity College Dublin
Steklov Mathematical Institute Moscow

Based on joint work arXiv:0807.4400 with L.F.ALDAY, G.ARUTYUNOV

$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories (N, k)



$\text{AdS}_4 \times S^7 / \mathbb{Z}_k$ solution of 11D supergravity

$$S^7 \xrightarrow{\pi} \mathbb{CP}^3; \quad \pi^{-1}(x) \sim S^1, \quad x \in \mathbb{CP}^3$$

Discrete
values of
the
coupling
constant

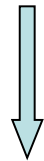
't Hooft limit $N, k \rightarrow \infty$, $\lambda \sim \frac{N}{k}$ fixed and real

IIA superstring theory on $\text{AdS}_4 \times \mathbb{CP}^3$

String theory on $\text{AdS}_4 \times \mathbb{CP}^3$ as a sigma model on the coset $\frac{\text{OSP}(2, 2|6)}{\text{SO}(3, 1) \times \text{U}(3)}$ maximally symmetric space

Maximal bosonic subgroup $\text{USP}(2, 2) \times \text{SO}(6)$

24 real supercharges



κ -symmetry fixing

16 real supercharges = # of physical bosonic d.o.f.

There exists a \mathbb{Z}_4 -grading of the Lie algebra $\mathfrak{osp}(2, 2|6)$

Let $g \in OSP(2, 2|6)$

Define a left-invariant $\mathfrak{osp}(2, 2|6)$ -valued 1-form $A \equiv g^{-1}dg$

Its decomposition under the grading gives

$A^{(2)}$ - vielbein (zehnbein)

$A^{(0)}$ - spin connection

$A^{(1)}, A^{(3)}$ - fermionic components

The Lagrangian

$$\mathcal{L} = \gamma^{\alpha\beta} \text{str}(A_{\alpha}^{(2)} A_{\beta}^{(2)}) + \kappa \epsilon^{\alpha\beta} \text{str}(A_{\alpha}^{(1)} A_{\beta}^{(3)})$$

$$\kappa = \pm 1 \quad \text{by } \kappa\text{-symmetry}$$

Coset parameterization

$$g = g_A g_\chi g_B$$



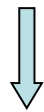
fermionic elements

Global symmetry group acts from the left $g \rightarrow g_0 g$, $g_0 \in \text{OSP}(2, 2|6)$

whereas local \mathcal{K} -symmetry acts from the right $g \rightarrow g e^\epsilon$, ϵ constrained

ARUTYUNOV, FROLOV 06''08'

g_A represents some submanifold in the coset,
on which group $\mathbb{H} \in \text{OSP}(2, 2|6)$ multiplication can be defined



Fermions χ are neutral under \mathbb{H}

The spinning string

Two non-zero Noether conserved charges (S, J)
with zero Poisson bracket (Cartan)

S - the spin in AdS_4

J - the spin in \mathbb{CP}^3

A classical solution representing a string rotating in AdS_4
with its baricenter moving along a circle in \mathbb{CP}^3

The limit $S, J \rightarrow \infty, \frac{J}{\log S}$ fixed

$$\Delta - S \equiv f\left(\lambda, \frac{J}{\log S}\right) \log S + \dots$$

The spinning string describes movement in $\text{AdS}_3 \times S^1$
and, as such, coincides with the solution found by

FROLOV,
TSEYTLIN 04''02'

$$t = \kappa\tau, \quad \phi = \omega_1\tau, \quad \varphi = \omega_2\tau, \quad \rho = \rho(\sigma)$$

Coset element for this solution

$$g = \begin{pmatrix} e^{\frac{i}{2}\kappa\tau\Gamma^0 - \frac{1}{2}\omega_1\tau\Gamma^2\Gamma^3} e^{-\frac{i}{2}\rho\Gamma^3} & 0 \\ 0 & e^{-\frac{1}{2}\omega_2\tau(T_{34}+T_{56})} e^{\frac{\pi}{4}T_5} \end{pmatrix}$$

The long string approximation

$$\rho' \simeq \text{const.}, \quad \max(\rho) \rightarrow \infty$$

Expand the action up to quadratic order in fermionic and bosonic fluctuations



Obtain the frequencies

Bosonic

$$m = \omega_2$$

$$4 \times m = \frac{\omega_2}{2}$$

$$m = \sqrt{2\kappa^2 - \omega_2^2}$$

$$\Omega_n^\pm = \sqrt{n^2 + 2\kappa^2 \pm 2\sqrt{\kappa^4 + n^2\omega_2^2}}$$

Fermionic

$$4 \times m = \kappa$$

$$2 \times \Omega_n^\pm = \sqrt{n^2 + \frac{1}{2}\kappa^2 \pm \frac{1}{2}\sqrt{\kappa^4 + 4\omega_2^2 n^2}}$$

The result

ALDAY,
ARUTYUNOV,
DB 07''08'

$$E_0 = \frac{J}{u} \qquad \delta E \sim \langle 0|H|0\rangle$$


$$\delta E = \frac{\omega_1}{4} \left[-2u^2 \log u^2 - 2 \log (8 - 4u^2) + u^2 \log (16(2 - u^2)) + \right. \\ \left. + 2 \left(-1 + u^2 + \sqrt{1 - u^2} + (u^2 - 2) \log (1 + \sqrt{1 - u^2}) \right) \right]$$

where

$$u = \frac{1}{\sqrt{1 + \left(\frac{\sqrt{\lambda}}{\pi J} \log \frac{S}{J}\right)^2}}$$

A limit

$$J = 0 \leftrightarrow u = 0 \qquad \delta E = -\frac{5 \log 2}{2\pi} \log S$$

Integrability of the superstring in $\text{AdS}_4 \times \mathbb{CP}^3$

- Classically integrable sigma model
- Two-loop dilatation operator known (in the scalar sector) and is integrable MINAHAN, ZAREMBO 06''08'
- \mathbb{CP}^n models not integrable at the quantum level ABDALLA, FORGER, GOMES 82'

The discrepancy

$$\delta E^{\text{AdS}_4 \times \mathbb{CP}^3} - \frac{1}{2} \delta E^{\text{AdS}_5 \times S^5} = \omega_1 (u^2 - 1) \log 2$$
$$\stackrel{?}{=} 0$$

GROMOV,
VIEIRA 07''08'

Can be resolved by tuning the dispersion relation

$$\epsilon(p) = \sqrt{1 + 16h(\lambda)^2 \sin^2\left(\frac{p}{2}\right)}$$

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + \underline{\text{const.}} + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

$\neq 0$

Conclusions

- obtained 1-loop correction to the energy of the spinning string in the long-string approximation and $S \sim e^J$
- discrepancy between our result and the prediction of the asymptotic Bethe ansatz
- discrepancy is removed if one modifies the dispersion relation of the giant magnon