Semiclassical quantization of spinning strings in $\mathrm{AdS}_4 \times \mathbb{CP}^3$

DMITRI BYKOV

Trinity College Dublin
Steklov Mathematical Institute Moscow

Based on joint work arXiv:0807.4400 with L.F.ALDAY, G.ARUTYUNOV

AHARONY, BERGMANN, JAFFERIS, MALDACENA 06"08'

$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories (N, k)

Discrete values of the coupling constant

$\Updownarrow$

$\mathrm{AdS}_4 \times \mathrm{S}^7/\mathbb{Z}_k$ solution of 11D supergravity

$$\mathrm{S}^7 \xrightarrow{\pi} \mathbb{C}\mathrm{P}^3; \quad \pi^{-1}(x) \sim S^1, \;\; x \in \mathbb{C}\mathrm{P}^3$$

't Hooft limit $N, k \to \infty, \quad \lambda \sim \dfrac{N}{k}$ fixed and real

IIA superstring theory on $\mathrm{AdS}_4 \times \mathbb{C}\mathrm{P}^3$

String theory on $AdS_4 \times \mathbb{C}P^3$
as a sigma model on the coset $\frac{OSP(2,2|6)}{SO(3,1) \times U(3)}$ ← maximally symmetric space

Maximal bosonic subgroup $USP(2,2) \times SO(6)$

24 real supercharges

↓ κ -symmetry fixing

16 real supercharges = # of physical bosonic d.o.f.

There exists a $\mathbb{Z}_4$-grading of the Lie algebra $\mathrm{osp}(2,2|6)$

Let $g \in OSP(2,2|6)$

Define a left-invariant $\mathrm{osp}(2,2|6)$-valued 1-form $\mathrm{A} \equiv g^{-1}dg$

Its decomposition under the grading gives

$\mathrm{A}^{(2)}$ - vielbein (zehnbein)

$\mathrm{A}^{(0)}$ - spin connection

$\mathrm{A}^{(1)},\ \mathrm{A}^{(3)}$ - fermionic components

The Lagrangian

$$\mathcal{L} = \gamma^{\alpha\beta}\mathrm{str}\big(A_\alpha^{(2)}A_\beta^{(2)}\big) + \kappa\epsilon^{\alpha\beta}\mathrm{str}\big(A_\alpha^{(1)}A_\beta^{(3)}\big)$$

$\kappa = \pm 1$ by κ-symmetry

Coset parameterization

$$g = g_{\mathrm{A}} g_{\chi} g_{\mathrm{B}}$$

↑ fermionic elements

Global symmetry group acts from the left $g \to g_0 g, \ g_0 \in \mathrm{OSP}(2,2|6)$

whereas local κ -symmetry acts from the right $g \to g e^{\epsilon}, \ \epsilon$ constrained

ARUTYUNOV, FROLOV 06"08'

g_{A} represents some submanifold in the coset,
on which group $\mathrm{H} \in \mathrm{OSP}(2,2|6)$ multiplication can be defined

⇓

Fermions χ are neutral under H

The spinning string

GUBSER, KLEBANOV, POLYAKOV 04"02'
FROLOV, TSEYTLIN 04"02'

Two non-zero Noether conserved charges (S, J)
with zero Poisson bracket (Cartan)

S - the spin in AdS_4

J - the spin in $\mathbb{C}\mathrm{P}^3$

A classical solution representing a string rotating in AdS_4
with its baricenter moving along a circle in $\mathbb{C}\mathrm{P}^3$

The limit $S, J \to \infty, \; \dfrac{J}{\log S}$ fixed

$$\Delta - S \equiv f(\lambda, \frac{J}{\log S}) \log S + \ldots$$

The spinning string describes movement in $\mathrm{AdS}_3 \times S^1$ and, as such, coincides with the solution found by **FROLOV, TSEYTLIN 04"02'**

$$t = \kappa\tau,\ \phi = \omega_1\tau,\ \varphi = \omega_2\tau,\ \rho = \rho(\sigma)$$

Coset element for this solution

$$g = \begin{pmatrix} e^{\frac{i}{2}\varkappa\tau\,\Gamma^0 - \frac{1}{2}\omega_1\tau\,\Gamma^2\Gamma^3} e^{-\frac{i}{2}\rho\,\Gamma^3} & 0 \\ 0 & e^{-\frac{1}{2}\omega_2\tau(T_{34}+T_{56})} e^{\frac{\pi}{4}T_5} \end{pmatrix}$$

The long string approximation

$$\rho' \simeq \text{const.},\ \max(\rho) \to \infty$$

Expand the action up to quadratic order in fermionic and bosonic fluctuations

⇓

Obtain the frequencies

Bosonic

$$m = \omega_2$$

4 x $$m = \frac{\omega_2}{2}$$

$$m = \sqrt{2\kappa^2 - \omega_2^2}$$

$$\Omega_n^{\pm} = \sqrt{n^2 + 2\varkappa^2 \pm 2\sqrt{\varkappa^4 + n^2\omega_2^2}}$$

Fermionic

4 x $$m = \kappa$$

2 x $$\Omega_n^{\pm} = \sqrt{n^2 + \frac{1}{2}\varkappa^2 \pm \frac{1}{2}\sqrt{\varkappa^4 + 4\omega_2^2 n^2}}$$

The result

ALDAY,
ARUTYUNOV,
DB 07"08'

$$E_0 = \frac{J}{u}$$

$$\delta E \sim \langle 0|H|0\rangle$$

⇓

$$\delta E = \frac{\omega_1}{4}\Big[-2u^2\log u^2 - 2\log(8-4u^2) + u^2\log(16(2-u^2)) +$$
$$+2\left(-1+u^2+\sqrt{1-u^2}+(u^2-2)\log(1+\sqrt{1-u^2})\right)\Big]$$

where

$$u = \frac{1}{\sqrt{1+(\frac{\sqrt{\lambda}}{\pi J}\log\frac{S}{J})^2}}$$

A limit

$$J = 0 \leftrightarrow u = 0 \qquad\qquad \delta E = -\frac{5\log 2}{2\pi}\log S$$

Integrability of the superstring in $\mathrm{AdS}_4 \times \mathbb{CP}^3$

- Classically integrable sigma model

- Two-loop dilatation operator known (in the scalar sector) and is integrable

MINAHAN, ZAREMBO 06"08'

- $\mathbb{CP}^n$ models not integrable at the quantum level

ABDALLA, FORGER, GOMES 82'

The discrepancy

$$\delta E^{\mathrm{AdS}_4\times\mathbb{CP}^3} - \frac{1}{2}\delta E^{\mathrm{AdS}_5\times\mathrm{S}^5} = \omega_1(u^2-1)\log 2$$

$$\stackrel{?}{=} 0$$

GROMOV,
VIEIRA 07"08'

Can be resolved by tuning the dispersion relation

$$\epsilon(p) = \sqrt{1+16h(\lambda)^2\sin^2(\frac{p}{2})}$$

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + \underline{\mathrm{const.}} + O(\frac{1}{\sqrt{\lambda}})$$

$$\neq 0$$

Conclusions

- obtained 1-loop correction to the energy of the spinning string in the long-string approximation and $S \sim e^J$
- discrepancy between our result and the prediction of the asymptotic Bethe ansatz
- discrepancy is removed if one modifies the dispersion relation of the giant magnon