A model of composite structure of quarks and leptons

N.G.Marchuk

December 2, 2002

Steklov Mathematical Institute, Gubkina st.8, Moscow 119991, Russia
nmarchuk@mi.ras.ru, www.orc.ru/˜nmarchuk

PACS: 10, 11.15, 11.30, 12.10, 12.60

Abstract

In the model every quark or lepton is identified with a quartet of four "more elementary" particles. One particle in a quartet is a massive spin-0 boson and other three particles are massless spin-1/2 fermions.

In the Standard Model leptons and quarks of three generations

$$\begin{pmatrix} \nu_e & u \\ e^- & d \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu & c \\ \mu^- & s \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau & t \\ \tau^- & b \end{pmatrix}. \tag{1}$$

are considered as fundamental (noncomposite) particles. In our model every quark or lepton is identified with a quartet of four "more elementary" particles. One particle in a quartet is a massive spin-0 boson and other three particles are massless spin-1/2 fermions. The first iteration of the model [1] was proposed as an attempt to improve Harari-Shupe model [5]-[8]. Our model has some common features with the Terazawa-Chikashige-Akama model [9, 10].

Let us assume the existence of ten elementary particles, which we divide in five groups

$$\beta^r, \beta^y, \beta^b; \quad \lambda; \quad \varepsilon^u, \varepsilon^d; \quad \delta; \quad \Delta^1, \Delta^2, \Delta^3.$$

Following [1, 2], we call them *inds*. We associate letters $\beta, \lambda, \varepsilon$ in designations of inds with the words "barion","lepton","electroweak" respectively. Superscripts r, y, b indicate (QCD) colors of inds $\beta^r, \beta^y, \beta^b$; superscripts u, d show the properties upness and downness of inds $\varepsilon^u, \varepsilon^d$ that associate with u, d quarks and with neutrino and electron; superscripts 1,2,3 of inds $\Delta^1, \Delta^2, \Delta^3$ indicate a generation of a particle. The quantum numbers of inds are collected in Table 1.

Table 1.

Ind	Q	B	L	color	spin	mass	suit
β	0	1/3	0	r, y, b	1/2	0	$\diamondsuit$
λ	$-2/3$	0	1	0	1/2	0	$\diamondsuit$
ε^u	2/3	0	0	0	1/2	0	$\heartsuit$
ε^d	$-1/3$	0	0	0	1/2	0	$\heartsuit$
δ	0	0	0	0	1/2	0	$\spadesuit$
Δ	0	0	0	0	0	m_1, m_2, m_3	$\clubsuit$

where Q - electric charge in units of proton charge; B - barion number; L - lepton number; color - QCD color charge. I don't know the real values of masses m_1, m_2, m_3, but I suppose that $m_1 < m_2 < m_3$. A suit is a new quantum number, which select admissible quartets of inds from all 715 quartets of inds. Namely quartets with full set of suits ($\diamondsuit\heartsuit\spadesuit\clubsuit$) we identify with fundamental fermions (1): the quartets $\beta, \varepsilon, \delta, \Delta$ (18 pieces) we identify with quarks and the quartets $\lambda, \varepsilon, \delta, \Delta$ (6 pieces) we identify with leptons.

Table 2.

ff	quartet	Q	B	L	color
ν_e	$\lambda\varepsilon^u\delta\Delta^1$	0	0	1	0
e^-	$\lambda\varepsilon^d\delta\Delta^1$	-1	0	1	0
u	$\beta\varepsilon^u\delta\Delta^1$	2/3	1/3	0	r, y, b
u	$\beta\varepsilon^d\delta\Delta^1$	$-1/3$	1/3	0	r, y, b

We get tables for fundamental fermions of second and third generations by replacing ind Δ^1 by Δ^2 and Δ^3 respectively. Fermion inds $\beta, \lambda, \varepsilon, \delta$ are massless and we may consider left-handed and right-handed inds as different particles. We suppose that left-handed inds $\varepsilon^u_L, \varepsilon^d_L$ have the weak isospin $I^w = 1/2$. All other inds have $I^w = 0$. In the following table an electric charge Q connected with a weak hypercharge Y^w and with a third component of weak isospin by the Gell-Mann–Nishijima formula $Q = I^w_3 + Y^w/2$

Table 3.

ind	Q	Y^w	I^w	I^w_3
ε^u_L	$2/3$	$1/3$	$1/2$	$1/2$
ε^u_R	$2/3$	$4/3$	0	0
ε^d_L	$-1/3$	$1/3$	$1/2$	$-1/2$
ε^d_R	$-1/3$	$-2/3$	0	0

Using the principles of Standard Model, we can write down the fundamental Lagrangian $\mathcal{L}_{\text{inds}}$ (similar to that of [2]), which describes a dynamics of particles on inds level. In this Lagrangian we must use wave functions of inds

$$|\beta^r\beta^y\beta^b\rangle, |\varepsilon^u_L\varepsilon^d_L\rangle, |\varepsilon^u_R\rangle, |\varepsilon^d_R\rangle, |\lambda\rangle, |\delta\rangle, |\Delta^k\rangle,\ k = 1, 2, 3.$$

Note that $|\beta^r\beta^y\beta^b\rangle$ is an SU(3)-triplet, $|\varepsilon^u_L\varepsilon^d_L\rangle$ is an U(1)$\times$SU(2)-doublet, $|\varepsilon^u_R\rangle, |\varepsilon^d_R\rangle, |\lambda\rangle$-are U(1)-singlets, and $|\delta\rangle, |\Delta^k\rangle$ are singlets without gauge symmetries. Wave functions of fermion inds satisfy Dirac equations and wave functions of boson inds satisfy Klein-Gordon equations.

According to the principles of Standard Model, we consider fundamental bosons – $\gamma, Z^0, W^\pm, g_1, \ldots, g_8$ (g_k -gluons of QCD) as gauge fields of $\mathcal{L}_{\text{inds}}$ relevant to U(1), U(1)$\times$SU(2)$_L$, SU(3) gauge symmetries.

From the other side we may consider fundamental bosons – $\gamma, Z^0, W^\pm$, $g_1, \ldots, g_8$ as composite particles that consist of ind-antiind pairs (superpositions of pairs) with $\diamondsuit\bar{\diamondsuit}$ or $\heartsuit\bar{\heartsuit}$ suits. Namely

$$\begin{aligned}
W^+ &= \varepsilon^u\bar{\varepsilon}^d, \\
W^- &= \bar{\varepsilon}^u\varepsilon^d, \\
Z^0 &= \frac{1}{\sqrt{2}}(\varepsilon^u\bar{\varepsilon}^u - \varepsilon^d\bar{\varepsilon}^d)\cos\Theta_W + \lambda\bar{\lambda}\sin\Theta_W, \\
\gamma &= -\frac{1}{\sqrt{2}}(\varepsilon^u\bar{\varepsilon}^u - \varepsilon^d\bar{\varepsilon}^d)\sin\Theta_W + \lambda\bar{\lambda}\cos\Theta_W, \\
g^{y\bar{b}} &= \beta^y\bar{\beta}^b, \ldots,
\end{aligned}$$

where Θ_W is the Weinberg angle

$$\cos\Theta_W = \frac{m_W}{m_Z}.$$

From this point of view we can easily interpret strong and electroweak interactions as inds exchange processes. For example

$$\begin{aligned}
u^r + g^{y\bar{r}} \longleftrightarrow u^y &\equiv \beta^r\,\varepsilon^u\,\delta\Delta^1 + \beta^y\,\bar{\beta}^r \longleftrightarrow \beta^y\,\varepsilon^u\,\delta\Delta^1 \\
g^{r\bar{b}} + g^{b\bar{y}} \longleftrightarrow g^{r\bar{y}} &\equiv \beta^r\,\bar{\beta}^b + \beta^b\,\bar{\beta}^y \longleftrightarrow \beta^r\,\bar{\beta}^y \\
e^- + W^+ \longleftrightarrow \nu_e &\equiv \lambda\,\varepsilon^d\,\delta\Delta^1 + \varepsilon^u\,\bar{\varepsilon}^d \longleftrightarrow \lambda\,\varepsilon^u\,\delta\Delta^1 \\
d + W^+ \longleftrightarrow u &\equiv \beta\,\varepsilon^d\,\delta\Delta^1 + \varepsilon^u\,\bar{\varepsilon}^d \longleftrightarrow \beta\,\varepsilon^u\,\delta\Delta^1
\end{aligned}$$

There is an evident analogy between the present model, which consider quarks and leptons as quartets of inds, and quantum chromodynamics, which consider barions as trios of quarks. This analogy leads us to the assumption that all inds have one more quantum number with four possible values, which we call *4-color*. Suppose that quarks and leptons are 4-color singlets (states antisymmetric in 4-color). Suppose also an existence of 4-color gluons, which are quanta of 4-color interaction that confines quartets of inds in quarks and leptons. It seems natural to describe a 4-color interaction as a gauge field theory with SU(4) symmetry (15 generators) that corresponds to 4-color.

So in the model we use gauge groups U(1),U(1) $\times$ SU(2),SU(3),SU(4), which are subgroups of the group U(4). According to [3, 4], for these gauge groups Dirac-type tensor equations can be used instead of Dirac equations.

References

[1] N.G.Marchuk, Doklady of Russian Academy of Science 344, 1 (1995).

[2] N.G.Marchuk, A model of the composite structure of quarks and leptons with SU(4) gauge symmetry, http://xxx.lanl.gov/abs/hep-ph/9801382.

[3] N.G.Marchuk, Nuovo Cimento, 117B, N.1, (2002), p.95.

[4] N.G.Marchuk, Dirac-type tensor equations on parallelisable manifolds, Nuovo Cimento, 117B, (2002), to appear.

[5] M.A.Shupe, Phys. Lett. 86B (1979) 87.

[6] H.Harari, Phys. Lett. 86B (1979) 83.

[7] H.Harari and N.Seiberg, Phys. Lett. 98B (1981) 269.

[8] H.Harari and N.Seiberg, Phys. Lett. 100B (1981) 41.

[9] H.Terazawa, Y.Chikashige and K.Akama, Phys. Rev. D15 (1977) 480.

[10] H.Terazawa, Phys. Rev. D22 (1980) 184.