1 Suppose $A, B \in \mathsf{NP}$. Is it true that $A \cup B \in \mathsf{NP}$? Is it true that $A \cap B \in \mathsf{NP}$?

2 Recall that the Kleene star A^* is a language consisting of words in the form

$$a_1 a_2 \ldots a_k, \quad \text{where } a_i \in A,\ k \geq 0.$$

Prove that if $A \in \mathsf{NP}$ then $A^* \in \mathsf{NP}$.

3 Show that if $\mathsf{P} = \mathsf{NP}$ then every language $A \in \mathsf{P}$ is NP-complete, except $A = \varnothing$ and $A = \Sigma^*$.

4 Show that for every polynomial time Turing machine M and polynomial p the language

$$\{x \mid \exists y \text{ s.t. } |y| \leq p(|x|) \text{ and } M(x,y) = 1\}$$

is in NP.

5 Suppose that $\mathsf{P} = \mathsf{NP}$. Prove that in this case there is a polynomial-time algorithm that *finds* a satisfying assignment to a given CNF.

6 **Extra.** Prove that if there is an NP-complete unary language then $\mathsf{P} = \mathsf{NP}$.

Problems for homework **Due: September, 29, 2017**

1 Double SAT problem.
Instance: a circuit ϕ.
Question: does ϕ have at least two satysfying assignments?
Prove that Double SAT problem is NP-complete.

2 Halting problem.
Instance: a description of a Turing machine M and its input w.
Question: does M halt on the input w?
Prove that the halting problem is NP-hard.