1 A clique in an undirected graph G is a subset of pairwise connected vertices.
CLIQUE problem: given a graph G and an integer k decide whether there is a clique in G of size at least k.
Show that CLIQUE is NP-complete.

2 *A vertex cover* of a graph G is a subset S of vertices of G such that each edge of G has at least one endpoint in S.
VERTEX COVER problem: given G and an integer k decide whether there is a vertex cover of size at most k in G.
Show that VERTEX COVER is NP-complete.

3 For arbitrary k consider a problem given a graph G to decide whether G contains a clique of size at least k. Show that for any k this problem is in P.

4 Prove that the following problem is NP-complete.
Set-partition problem.
Instance: A finite sequence of $x_1, \dots, x_n$ of positive integers.
Question: is there a set $I \subset \{1, \dots, n\}$ such that $\sum_{i \in I} x_i = \sum_{i \notin I} x_i$?

5 Let G be a graph. A clique partition of G is a family of disjoint subsets of the vertex set of the graph such that (i) each subset induces a clique and (ii) any vertex of the graph is contained in some subset.
The clique partition problem: given a graph G and an integer k decide whether G has a clique partition of size at most k?
Prove that the clique partition problem is NP-complete.

6 Let G be a graph. A clique cover of G is a family of subsets (not necessarily disjoint) of the vertex set of the graph such that (i) each subset induces a clique and (ii) any vertex of the graph is contained in some subset.
The clique cover problem: given a graph G and an integer k decide whether G has a clique cover of size k?
Prove that the clique cover problem is NP-complete.

Problems for homework **Due: September, 29, 2017**

1 *A dominating set* is a subset U of vertices of a graph G such that every other vertex in G is adjacent to some vertex in U.
DOMINATING-SET problem.
Instance: a graph G, an integer k.
Question: is there a dominating set of size k in the graph G?
Prove that the DOMINATING-SET problem is NP-complete.

2 The SUBSET PROD problem.
Instance: a list of integers $a_1, \dots, a_n, b$.
Question: is there a subset $S \subseteq \{1, 2, \dots, n\}$ such that

$$\prod_{i \in S} a_i = b \ ?$$

Prove that the SUBSET PROD problem is NP-complete.
Warning: exponentiation does not reduce the SUBSET SUM to the SUBSET PROD. (Why?)
Hint: Assume the prime number theorem:

$$\pi(n) \sim \frac{n}{\ln n},$$

where $\pi(n)$ is the number of primes in the range $2, \dots, n$, and $\ln n$ is the natural logarithm.