1 Prove that if $\mathsf{NP} \neq \mathsf{coNP}$, then $\mathsf{P} \neq \mathsf{NP}$.

2 Show that if $\mathsf{NP} \subseteq \mathsf{coNP}$ then $\mathsf{NP} = \mathsf{coNP}$.

3 Prove that if $\mathsf{EXP} \neq \mathsf{NEXP}$, then $\mathsf{P} \neq \mathsf{NP}$.

4 Show that TQBF restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.

5 Show that if $TQBF \in \mathsf{NP}$, then $\mathsf{NP} = \mathsf{coNP}$.

6 The cat-and-mouse game is played by two players, "Cat" and "Mouse" on an arbitrary graph. At a given moment of time each player occupies a vertex of a graph. Players move in turn. A player is allowed to move in any vertex adjacent to the current position.
A special vertex of the graph is called Hole.
Cat wins if the two players ever occupy the same node. Mouse wins if it reaches the Hole before the preceding happens. The game is draw if a situation repeats (a situation is determined by player's positions and player's turn to move).
The Happy Cat problem.
Instance: a graph G, vertices c, m, h of the G (initial positions of Cat and Mouse, the Hole).
Question: has Cat a winning strategy if Cat moves first?
Prove that the Happy Cat problem is in P.

A language is called *unary* if it is a subset of $\{1\}^*$.

7 Prove that if every unary NP-language is in P then $\mathsf{EXP} = \mathsf{NEXP}$.

8 Consider TQBF problem restricted to monotone formulas (no negations). Show (at least) one of the two: it is in P; it is PSPACE-complete.

9 Prove that the following problem is PSPACE-complete.
SUCCINCT CONNECTIVITY problem.
Given a directed graph G represented by a Boolean circuit (that is, there is a Boolean circuit given which inputs are considered as the numbers of two vertices and the output is 1 if there is an edge between them) and two nodes s and t decide whether there is a path from s to t in G.

10 Extra. Show that $\mathsf{P} \neq \mathsf{SPACE}(n)$. (Hint: show first that if $\mathsf{SPACE}(n) \subseteq \mathsf{P}$ then $\mathsf{P} = \mathsf{PSPACE}$.)

Problems for homework **Due: September, 29, 2017**

1 Suppose $L_1, L_2 \in \mathsf{NP} \cap \mathsf{coNP}$. Show that $L_1 \oplus L_2 = \{x : x \text{ is in exactly one of } L_1, L_2\}$ is in $\mathsf{NP} \cap \mathsf{coNP}$.

2 The problem IN-SPACE ACCEPTANCE.
Instance: a Turing machine M, an input x.
Question: Does the TM M accept x without ever leaving the first $|x|$ cells on the tape?
(a)Prove that the problem IN-SPACE ACCEPTANCE is in PSPACE.
(b) Prove that the problem IN-SPACE ACCEPTANCE is PSPACE-complete.