Colloquium program for Theory of Computing 2018

In the beginning of colloquium you will draw two questions, one from each part of the program below. You will have about an hour to prepare. You are not allowed to use any materials during the colloquium. When you are ready you can give your answers to one of the instructors.

The grade is formed in the following way. You get 2 points just for showing up to the colloquium. You get 2 points for presenting the most basic knowledge of the course. This will be checked only if you fail part 1 and part 2 of the colloquium; by default we assume that you know this. You get 3 points for each of the questions from Part 1 and Part 2. Part 2 is supposed to contain harder questions. You can get partial credit for partial answers.

Basic Knowledge

1. Definition of the complexity class P.
2. Definition of the complexity class NP.
3. The notion of polynomial time reduction.
4. The notion of NP-completeness.
5. An example of an NP-complete problem.
6. Definition of the complexity class PSPACE.
7. Definition of the complexity class BPP.
8. Definition of the complexity classes NC^i.
9. Definition of deterministic communication complexity.

Part 1

1. Complexity classes P and EXP. The complexity class NP: two definitions and their equivalence. Examples. Inclusions between P, NP and EXP.

2. Polynomial reductions, their properties. NP-hardness and NP-completeness, their properties.

3. Space complexity. Classes PSPACE and $\mathsf{NPSPACE}$. Configuration graph. Inclusions between time and space classes.

4. Classes L and NL. Examples. Log-space reductions, their properties.

5. Probabilistic computation. Probabilistic machines, the class BPP. Invariance of the definition BPP for different thresholds. BPP is in $\mathsf{P}/poly$.

6. $\mathsf{ZPP} = \mathsf{RP} \cap \mathsf{coRP}$.

7. Computations with oracles, simple properties.

8. Complexity classes NC^i and AC^i. Directed reachability is in AC^1. Characterization of NC^0.

9. Relation of NC^i and AC^i to L and NL.

10. Streaming algorithms. Finding the majority element.

11. Communication protocols. Fooling sets. Combinatorial rectangles. Deterministic communication complexity of EQ, GT, DISJ.

12. Rectangle size lower bound. Deterministic communication complexity of IP. Rank lower bound.

13. Non-deterministic complexity. Non-deterministic communication complexity of EQ, GT, DISJ, IP.

14. Randomized communication complexity, definitions. $\mathsf{R}^{\text{pub}}(\text{EQ}) = O(1)$ and $\mathsf{R}^{\text{priv}}(\text{EQ}) = O(\log n)$. $\mathsf{N}^1(f)$ vs. $\mathsf{R}^{1,\text{priv}}(f)$.

15. Deterministic streaming algorithms that compute the maximal frequency exactly use space $\Omega(\min\{m, n\})$.

Part 2

1. Time and space hierarchy theorems.
2. NP-completeness: CIRC-SAT, 3-SAT, NAE-3-SAT.
3. NP-completeness: SUBSETSUM.
4. NP-completeness: IND-SET, 3-COL.
5. TQBF problem, its PSPACE-completeness. $\mathsf{PSPACE} = \mathsf{NPSPACE}$.
6. Polynomial randomized primality test.
7. REACHABILITY is NL-complete.
8. There are oracles A and B such that $\mathsf{P}^A = \mathsf{NP}^A$ and $\mathsf{P}^B \neq \mathsf{NP}^B$.
9. Addition is in AC^0. Multiplication is in NC^1.
10. Polynomial size Boolean formulas equal NC^1. Communication complexity: the minimal number of leafs in a protocol tree versus D.
11. Randomized approximate computation of the moment F_2 in logarithmic space.
12. $D(\text{MED}) = O(\log n)$ and $D(\text{CIS}) = O(\log^2 n)$, (CIS = clique-independent set problem).
13. $\mathsf{D}(f) = O(\mathsf{N}^0(f) \cdot \mathsf{N}^1(f))$.
14. The randomized one-shot complexity of DISJ is $\Omega(n)$.