**1.1** Suppose $A, B \in \mathsf{NP}$. Is it true that $A \cup B \in \mathsf{NP}$? Is it true that $A \cap B \in \mathsf{NP}$?

**1.2** Recall that the Kleene star $A^*$ is a language consisting of words in the form

$$a_1 a_2 \dots a_k, \quad \text{where } a_i \in A,\ k \geq 0.$$

Prove that if $A \in \mathsf{NP}$ then $A^* \in \mathsf{NP}$.

**1.3** Show that if $\mathsf{P} = \mathsf{NP}$ then every language $A \in \mathsf{P}$ is $\mathsf{NP}$-complete, except $A = \varnothing$ and $A = \Sigma^*$.

**1.4** Suppose that $\mathsf{P} = \mathsf{NP}$. Prove that in this case there is a polynomial-time algorithm that *finds* a satisfying assignment to a given CNF.

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**Problems for homework** **Due: October, 2, 2018**

**1.5** Show that for every polynomial time Turing machine $M$ and polynomial $p$ the language

$$\{x \mid \exists y \text{ s.t. } |y| \leq p(|x|) \text{ and } M(x, y) = 1\}$$

is in $\mathsf{NP}$.

**1.6** Let $\mathsf{NEXP} = \cup_{k \in \mathbb{N}} \mathsf{NTIME}(2^{n^k})$. Show that $L \in \mathsf{NEXP}$ iff there is a polynomial time Turing Machine $M$ and $k \in \mathbb{N}$ such that

$$x \in L \Leftrightarrow \exists y \in \{0,1\}^{2^{n^k}} \text{ such that } M(x, y) = 1.$$

**1.7** **Extra.** Prove that if there is an $\mathsf{NP}$-complete unary language then $\mathsf{P} = \mathsf{NP}$.