

3.1 A clique in an undirected graph G is a subset of pairwise connected vertices.

CLIQUE problem: given a graph G and an integer k decide whether there is a clique in G of size at least k .

Show that CLIQUE is NP-complete.

3.2 A vertex cover of a graph G is a subset S of vertices of G such that each edge of G has at least one endpoint in S .

VERTEX COVER problem: given G and an integer k decide whether there is a vertex cover of size at most k in G .

Show that VERTEX COVER is NP-complete.

3.3 For arbitrary k consider a problem given a graph G to decide whether G contains a clique of size at least k . Show that for any k this problem is in P.

Problems for homework

Due: October, 2, 2018

3.4 Double SAT problem.

Instance: a circuit ϕ .

Question: does ϕ have at least two satisfying assignments?

Prove that Double SAT problem is NP-complete.

3.5 Halting problem.

Instance: a description of a Turing machine M and its input w .

Question: does M halt on the input w ?

Prove that the halting problem is NP-hard.

3.6 Extra. 1-OR-3-SAT: given a 3-CNF $\phi(x)$ decide whether there is an assignment x such that in each clause there are 1 or 3 true literals. Show that 1-OR-3-SAT is in P