4.1 Let G be a graph. A clique partition of G is a family of disjoint subsets of the vertex set of the graph such that (i) each subset is a clique and (ii) any vertex of the graph is contained in some subset.
The clique partition problem: given a graph G and an integer k decide whether G has a clique partition of size at most k?
Prove that the clique partition problem is NP-complete.

4.2 Prove that if $\mathsf{NP} \neq \mathsf{coNP}$, then $\mathsf{P} \neq \mathsf{NP}$.

4.3 Show that if $\mathsf{NP} \subseteq \mathsf{coNP}$ then $\mathsf{NP} = \mathsf{coNP}$.

4.4 Prove that if $\mathsf{EXP} \neq \mathsf{NEXP}$, then $\mathsf{P} \neq \mathsf{NP}$.

4.5 Prove that the following problem is NP-complete.
Set-partition problem.
Instance: A finite sequence of $x_1, \ldots, x_n$ of positive integers.
Question: is there a set $I \subset \{1, \ldots, n\}$ such that $\sum_{i \in I} x_i = \sum_{i \notin I} x_i$?

Problems for homework **Due: October, 2, 2018**

4.6 Let G be a graph. A clique cover of G is a family of subsets (not necessarily disjoint) of the vertex set of the graph such that (i) each subset induces a clique and (ii) any vertex of the graph is contained in some subset.
The clique cover problem: given a graph G and an integer k decide whether G has a clique cover of size k?
Prove that the clique cover problem is NP-complete.

4.7 Suppose $L_1, L_2 \in \mathsf{NP} \cap \mathsf{coNP}$. Show that $L_1 \oplus L_2 = \{x : x \text{ is in exactly one of } L_1, L_2\}$ is in $\mathsf{NP} \cap \mathsf{coNP}$.

4.8 **Extra.** Show that $\mathsf{P} \neq \mathsf{SPACE}(n)$. (Hint: show first that if $\mathsf{SPACE}(n) \subseteq \mathsf{P}$ then $\mathsf{P} = \mathsf{PSPACE}$.)