5.1 Show that if TQBF $\in \mathsf{NP}$, then $\mathsf{NP} = \mathsf{coNP}$.

5.2 Show that TQBF restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.

5.3 Show that there exists a language in $\mathsf{SPACE}(n^3)$ that does not belong to $\mathsf{NSPACE}(n)$.

5.4 The cat-and-mouse game is played by two players, "Cat" and "Mouse" on an arbitrary graph. At a given moment of time each player occupies a vertex of a graph. Players move in turn. A player is allowed to move in any vertex adjacent to the current position. A special vertex of the graph is called Hole.

Cat wins if the two players ever occupy the same node. Mouse wins if it reaches the Hole before the preceding happens. The game is draw if a situation repeats (a situation is determined by player's positions and player's turn to move).

The Happy Cat problem.
Instance: a graph G, vertices c, m, h of the G (initial positions of Cat and Mouse, the Hole).
Question: has Cat a winning strategy if Cat moves first?

Prove that the Happy Cat problem is in P.

5.5 Consider TQBF problem restricted to monotone formulas (no negations). Show (at least) one of the two: it is in P; it is PSPACE-complete.

In the following problem, we interpret circuits with $2m$ inputs as a directed graph $G = (V, E)$: the vertices are $V = \{0, 1\}^m$, and there is an edge from x to y if and only if $\phi(xy) = 1$. Thus,

$$(x, y) \in E \iff \phi(xy) = 1.$$

5.6 SUCCINCT CONNECTIVITY problem.
Input: A circuit interpreted as a graph, two vertices s and t.
Question: Is there a path from s to t.

Prove that this problem is PSPACE-complete.

Problems for homework **Due: November 6, 2018**

5.7 The problem IN-SPACE ACCEPTANCE.
Instance: a Turing machine M, an input x.
Question: Does the TM M accept x without ever leaving the first $|x|$ cells on the tape?

(a) Prove that the problem IN-SPACE ACCEPTANCE is in PSPACE.

(b) Prove that the problem IN-SPACE ACCEPTANCE is PSPACE-complete.

5.8 The problem HITTING-SET.
Instance: sets $S_1, \ldots, S_e$, integer k.
Question: Does there exist a set of at most k elements that contains at least one element from every set S_i?

Prove that this problem is NP-complete.