**6.1**  Suppose that $A, B \in \mathsf{L}$. Show that $A \cap B, A \cup B \in \mathsf{L}$.

**6.2**  Suppose that $A, B \in \mathsf{NL}$. Show that $A \cap B, A \cup B \in \mathsf{NL}$.

**6.3**  Let $A$ be the language of properly nested parenthesis (in the alphabet $\{(,)\}$). Show that $A \in \mathsf{L}$.

**6.4**  Show that if $f, g \colon \{0,1\}^* \to \{0,1\}^*$ are log-space computable functions, then $g \circ f$ is also log-space computable.

**6.5**  Show that the directed reachability problem is $\mathsf{NL}$-complete even when restricted to acyclic graphs.

**6.6**  Let UCYCLE be the language of undirected graphs $G$ that has a cycle in them (note that $G$ does not have to be connected). Show that UCYCLE is in $\mathsf{L}$.

**6.7**  The cat-and-mouse game is played by two players, "Cat" and "Mouse" on an arbitrary graph. At a given moment of time each player occupies a vertex of a graph. Players move in turn. A player is allowed to move in any vertex adjacent to the current position. A special vertex of the graph is called Hole.

Cat wins if the two players ever occupy the same node. Mouse wins if it reaches the Hole before the preceding happens. The game is draw if a situation repeats (a situation is determined by player's positions and player's turn to move).

The Happy Cat problem.
Instance: a graph $G$, vertices $c, m, h$ of the $G$ (initial positions of Cat and Mouse, the Hole).
Question: has Cat a winning strategy if Cat moves first?

Prove that the Happy Cat problem is in $\mathsf{P}$.

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**Problems for homework**                                                    **Due: November 6, 2018**

**6.8**  Show that if $A \in \mathsf{NL}$, then $A^* \in \mathsf{NL}$.

**6.9**  Let BIPARTITE be the language of undirected bipartite graphs. Show that BIPARTITE is in $\mathsf{NL}$.

**6.10 Extra** Show that 2-SAT is $\mathsf{NL}$-complete.