

- 9.1 Show that addition of two  $n$ -bit numbers is in  $AC^0$ .
- 9.2 Show that multiplication of two  $n$ -bit numbers is in  $AC^1$ .
- 9.3 Show that multiplication of two  $n$ -bit numbers is in  $NC^1$ .
- 9.4 The majority function of  $n$  bits outputs 1 iff at least half of its inputs are equal to 1. Show that the majority function is in  $NC^1$ .
- 9.5 The input to the bit sorting problem is a sequence  $x_1, \dots, x_n$ , where each  $x_i \in \{0, 1\}$ . The goal is to output the same bits in the sorted order. Show that this problem is in  $NC^1$ .
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**Problems for homework****Due: December 4, 2018**

- 9.6 The function  $IP(x, y)$  computes the inner product of vectors  $x, y \in \{0, 1\}^n$  over the field  $\mathbb{F}_2$ . Show that  $IP \in NC^1$ .
- 9.7 The function  $GT(x, y)$  for  $x, y \in \{0, 1\}^n$  is equal to 1 iff  $x \geq y$  ( $x$  and  $y$  are considered as integers in binary representation). Show that  $GT \in AC^0$ .
- 9.8 **Extra** The input to the sorting problem is a sequence  $a_1, \dots, a_n$ , where each  $a_i \in \{0, 1\}^n$  is considered as an integer in binary representation. The goal is to output the same bits in the sorted order. Show that this problem is in  $NC^1$ .