Colloquium program for Theorist's Toolkit 2019

In the beginning of colloquium you will draw three questions, two from the first part and one from the second part of the program below. You will have about an hour to prepare. You are not allowed to use any materials during the colloquium. When you are ready you can give your answers to one of the instructors.

The grade is formed in the following way. You get 1 point just for showing up to the colloquium. You get 3 points for the full answer to each of the questions. You can get partial credit for partial answers.

Part 1

1. Fourier coefficients, Fourier representation of Boolean functions. Orthogonality of monomials. Parceval's theorem.

2. Formula for expectation via Fourier coefficients. Formulas for variance: general and Boolean functions.

3. Probability density, relation to convolution. Fourier coefficients of the convolution.

4. Influences and discrete derivatives, relations between them. Influences through Fourier coefficients.

5. Total influence. Majority maximizes total influence among monotone functions.

6. Total influence through Fourier coefficients. Relation between variation and the total influence.

7. Noise stability and noise sensitivity, formulas via Fourier coefficients. Noise operator. Dictators are the most stable among unbiased Boolean functions.

8. Low-degree spectral concentration, bounds through the total influence and noise sensitivity.

9. Indicators of linear and affine subspaces, their Fourier representation. Application to decision trees.

10. Restrictions, Fourier coefficients under restrictions.

11. Paley-Zygmund inequality. B-reasonability, its simple properties.

12. Threshold functions. Chow's parameters. Concentration on degree 1.

13. Polynomial threshold functions. Sparsity, lower and upper bounds.

14. Decision trees, sensitivity, block sensitivity, certificate complexity. Polynomial relation between these measures.

15. Decision tree complexity and degree are polynomially related.

16. Chebyshev polynomials, their basic properties. Polynomial of degree $\sqrt{n}$ approximating OR_n in l_∞ norm.

17. Simultaneous multi-party communication complexity. INDEX and SUM-INDEX functions, lower and upper bounds.

Part 2

1. Linearity testing.

2. Arrow's theorem. Upper bound on the success probability in Condorcet election with any transitive-symmetric rule.

3. PAC-model. Reduction of learning problem to finding a collection of sets, on which the Fourier spectrum is concentrated. Applications (via low-degree algorithm).

4. Goldreich-Levin algorithm.

5. The Bonami Lemma. Anti-concentration of low degree polynomials. FKN Theorem.

6. Approximation of $\mathsf{ACC}^0[2]$ by low-degree polynomials over $\mathbb{F}_3$.

7. PARITY cannot be approximated by low-degree polynomials over $\mathbb{F}_3$