2.1 Compute the Fourier expansion of $\varphi_{\{0\}}(x_1, \ldots, x_n)$.

2.2 For functions $f\colon \{-1,1\}^n \to \mathbb{R}$ and $i \in [n]$ define the linear operator

$$\mathrm{E}_i f(x) = \mathop{\mathbf{E}}_{\boldsymbol{x}_i}[f(x_1, \ldots, x_{i-1}, \boldsymbol{x}_i, x_{i+1}, \ldots, x_n)].$$

Show that
(a) $\mathrm{E}_i f(x) = \frac{f(x^{i\to 1})+f(x^{i\to -1})}{2}$;
(b) $\mathrm{E}_i f(x) = \sum_{S \not\ni i} \widehat{f}(S) x^S$;
(c) $f(x) = x_i \mathrm{D}_i f(x) + \mathrm{E}_i f(x)$.

2.3 For functions $f\colon \{-1,1\}^n \to \mathbb{R}$ and $i \in [n]$ define the linear operator

$$\mathrm{L}_i f = f - \mathrm{E}_i f.$$

Show that
(a) $\mathrm{L}_i f(x) = \frac{f(x)-f(x^{\oplus i})}{2}$;
(b) $\mathrm{L}_i f(x) = x_i \mathrm{D}_i f(x) = \sum_{S \ni i} \widehat{f}(S) x^S$;
(c) $\langle f, \mathrm{L}_i f\rangle = \langle \mathrm{L}_i f, \mathrm{L}_i f\rangle = \mathbf{Inf}_i[f]$.

2.4 The Laplacian operator L is the linear operator on functions $f\colon \{-1,1\}^n \to \mathbb{R}$ defined by $\mathrm{L} = \sum_{i=1}^n \mathrm{L}_i$. Show that for $f\colon \{-1,1\}^n \to \{-1,1\}$ we have
(a) $\mathrm{L}f(x) = f(x)\mathrm{sens}_f(x)$;
(b) $\mathrm{L}f = \sum_S |S| \widehat{f}(S) \chi_S$.

2.5 Show that $\mathbf{E}_{\boldsymbol{x}}[\mathrm{sens}_f(\boldsymbol{x})] = \mathbf{Inf}[f] = \mathbf{E}_{\boldsymbol{S}\sim\mathcal{S}_f}[|\boldsymbol{S}|]$.

2.6 Show that $\mathbf{E}_{\boldsymbol{x}}[\mathrm{sens}_f(\boldsymbol{x})^2] = \mathbf{E}_{\boldsymbol{S}\sim\mathcal{S}_f}[|\boldsymbol{S}|^2]$.

Problems for homework **Due: January, 31, 2018**

2.7 Compute the Fourier expansion of $\varphi_{\{a\}}(x_1, \ldots, x_n)$ for arbitrary $a \in \mathbb{F}^n$.

2.8 Let $f, g, h\colon \mathbb{F}^n \to \mathbb{R}$. Show that

$$f * (g * h) = (f * g) * h, \quad f * g = g * f.$$

2.9 Let $f\colon \{-1,1\}^n \to \{-1,1\}$. Show that for any $i \in [n]$ we have $\mathbf{Inf}_i[f] \le \mathbf{Var}[f]$.