4.1 For $f\colon \{-1,1\}^n \to \{-1,1\}$ show that $\mathbf{Inf}[f] \le \deg(f)$.

4.2 Let $f\colon \{-1,1\}^n \to \mathbb{R}$ and $J \subseteq [n]$. Define $f^{\subseteq J}\colon \{-1,1\}^n \to \mathbb{R}$ by $f^{\subseteq J}(x) = \mathbf{E}_{\boldsymbol{y}\sim\{-1,1\}^{\overline{J}}}[f(x_J, \boldsymbol{y})]$, where $x_j \in \{-1,1\}^J$ is the projection of x to coordinates J. Show that

$$f^{\subseteq J} = \sum_{S \subseteq J} \widehat{f}(S)\chi_S.$$

4.3 Suppose $\deg(f) \le k$ for $f\colon \{-1,1\}^n \to \mathbb{R}$. Show that if one of the subfunctions $f(x_1, ..., x_{n-1}, \pm 1)$ is identically 0, then the other has degree at most $k-1$.

4.4 Suppose $\deg(f) \le k$ for $f\colon \{-1,1\}^n \to \mathbb{R}$ and f is not identically 0. Show that $\Pr[f(\boldsymbol{x}) \neq 0] \ge 2^{-k}$.

4.5 If $f\colon \{-1,1\}^n \to \{-1,1\}$ has $\deg(f) \le k$ then $\mathbf{Inf}_i[f]$ is either 0 or at least 2^{1-k} for all $i \in [n]$.

4.6 Suppose $f\colon \{-1,1\}^n \to \{-1,1\}$ has $\deg(f) \le k$. Show that f is a $k2^{k-1}$-junta.

4.7 Show that for any k there is $f\colon \{-1,1\}^n \to \{-1,1\}$ with $\deg(f) \le k$ such that f is not a $\frac{2^k}{100}$-junta.

4.8 Let $f\colon \mathbb{F}^n \to \{-1,1\}$ be computable by a decision tree of size s and let $\varepsilon \in (0,1]$. Show that the spectrum of f is ε-concentrated on degree up to $O(\log(s/\varepsilon))$.

4.9 Let $\mathcal{C} = \{f\colon \{-1,1\}^n \to \{-1,1\} \mid \mathrm{DT}_{size}(f) \le s\}$. Then $\mathcal{C}$ is learnable from random examples with error ε in time $n^{O(\log(s/\varepsilon))}$.

Problems for homework **Due: February, 14, 2019**

4.10 For $f, g\colon \mathbb{F}^n \to \mathbb{R}$ Show that $\widehat{\|fg\|}_1 \le \widehat{\|f\|}_1 \widehat{\|g\|}_1$.

4.11 Consider $f\colon \{-1,1\}^n \to \{-1,1\}$. Show that $\mathrm{spar}(f) \le \mathrm{gran}(f)^2$.

4.12 Suppose $f\colon \{-1,1\}^n \to \mathbb{R}$ is computable by a decision tree that has a leaf at depth k labeled by b. Show that $\widehat{\|f\|}_\infty \ge |b|/2^k$.