5.1 For even n find a function $f\colon \{-1,1\}^n \to \{-1,1\}$ that is not $1/2$-concentrated on any $\mathcal{F} \subseteq 2^{[n]}$ with $|\mathcal{F}| < 2^{n-1}$.

5.2 Show that for any $f\colon \mathbb{F}_2^n \to \{-1,1\}$ we have $\operatorname{gran}(f) \le 2^{n-1}$.

5.3 Let $f\colon \mathbb{F}_2^n \to \{-1,1\}$ be computable by a decision tree of size s and let $\varepsilon \in (0,1]$. Show that the spectrum of f is ε-concentrated on degree up to $O(\log(s/\varepsilon))$.

5.4 Let $\mathcal{C} = \{f\colon \{-1,1\}^n \to \{-1,1\} \mid \mathrm{DT}_{size}(f) \le s\}$. Then $\mathcal{C}$ is learnable from random examples with error ε in time $n^{O(\log(s/\varepsilon))}$.

5.5 Suppose $f\colon \mathbb{F}_2^n \to \{-1,1\}$ has $\operatorname{spar}(f) < 2^n$. Show that for any $\gamma \in \operatorname{supp}(f)$ there exists non-zero $\beta \in \mathbb{F}_2^n$ such that $f_{\beta^\perp}$ has $\widehat{f}(\gamma)$ as a Fourier coefficient.

5.6 Suppose $f\colon \mathbb{F}_2^n \to \{-1,1\}$ has $\operatorname{spar}(f) = s > 1$. Show that $\operatorname{gran}(f) \le 2^{\lfloor \log s \rfloor - 1}$.

5.7 Show that the class $\mathcal{C} = \{f\colon \{-1,1\}^n \to \{-1,1\} \mid f \text{ is a } k\text{-junta}\}$ can be learned exactly (with error 0) using queries in time $\operatorname{poly}(n, 2^k)$.

5.8 Show that the class $\mathcal{C} = \{f\colon \{-1,1\}^n \to \{-1,1\} \mid \mathrm{DT}(f) \le k\}$ can be learned exactly (with error 0) using queries in time $\operatorname{poly}(n, 2^k)$.

5.9 Show that the class $\mathcal{C} = \{f\colon \{-1,1\}^n \to \{-1,1\} \mid \operatorname{spar}(\widehat{f}) \le 2^{O(k)}\}$ can be learned exactly (with error 0) using queries in time $\operatorname{poly}(n, 2^k)$.

Problems for homework **Due: February, 14, 2019**

5.10 Show that we can learn any function $f\colon \{-1,1\}^n \to \{-1,1\}$ with error 0 from random samples in time $\widetilde{O}(2^n)$ ($\widetilde{O}(f(n))$ means $O(f(n)\mathsf{polylog}(f(n)))$).

5.11 Suppose the Fourier spectrum of $f\colon \{-1,1\}^n \to \mathbb{R}$ is ε_1-concentrated on $\mathcal{F}$ and that $g\colon \{-1,1\}^n \to \mathbb{R}$ satisfies $\|f-g\|_2^2 \le \varepsilon_2$. Show that the Fourier spectrum of g is $2(\varepsilon_1+\varepsilon_2)$-concentrated on $\mathcal{F}$.

5.12 Let $f\colon \{-1,1\}^n \to \mathbb{R}$ and let $\varepsilon > 0$. Show that f is ε-concentrated on a collection $\mathcal{F} \subseteq 2^{[n]}$ with $|\mathcal{F}| \le \widehat{\|f\|}_1^2/\varepsilon$.