7.1 Suppose $f\colon \{-1,1\}^n \to \mathbb{R}$ is given by $g(x) = |\sum_{i=1}^n w_i x_i|$ for some $w_1, \ldots, w_n \in \mathbb{R}$. Show that $\mathrm{L}g(x) \le g(x)$ for any $x \in \{-1,1\}^n$ (recall that $\mathrm{L} = \mathrm{L}_1 + \ldots + \mathrm{L}_n$).

7.2 Show that if $f\colon \{-1,1\}^n \to \mathbb{R}$ is an even function, then $\mathbf{Var}[f] \le \frac{1}{2}\mathbf{Inf}[f]$.

7.3 Suppose $f\colon \{-1,1\}^n \to \mathbb{R}$ is given by $g(x) = |\sum_{i=1}^n w_i x_i|$ for some $w_1, \ldots, w_n \in \mathbb{R}$. Show that $2\,\mathbf{Var}[g] \le \mathbf{E}[g^2]$.

7.4 Suppose $f\colon \{-1,1\}^n \to \mathbb{R}$ is given by $g(x) = |\sum_{i=1}^n w_i x_i|$ for some $w_1, \ldots, w_n \in \mathbb{R}$. Show that $\mathbf{E}_{\boldsymbol{x}}[g(x)] \ge \frac{1}{\sqrt{2}}\,\mathbf{E}[g^2]^{1/2}$.

7.5 Suppose $l\colon \{-1,1\}^n \to \mathbb{R}$ is defined by $l(x) = a_0 + a_1x_1 + \ldots a_nx_n$. Define $\widetilde{l}\colon \{-1,1\}^{n+1} \to \mathbb{R}$ by $\widetilde{l}(x_0, x_1, \ldots, x_n) = a_0x_0 + a_1x_1 + \ldots a_nx_n$. Show that $\|\widetilde{l}\|_1 = \|l\|_1$ and $\|\widetilde{l}\|_2 = \|l\|_2$.

7.6 Suppose $f\colon \{-1,1\}^n \to \{-1,1\}$ is computable by a DNF with s terms. Show that f has a PTF representation of sparsity $O(ns^2)$.

Problems for homework **Due: February, 28, 2019**

7.7 Show that $f\colon \{-1,1\}^n \to \{-1,1\}$ is an indicator of a Hamming ball iff it is expressible as a linear threshold function $\mathrm{sgn}(a_0 + a_1x_1 + \ldots a_nx_n)$ with $|a_1| = \ldots = |a_n|$.

7.8 Let $f\colon \{-1,1\}^n \to \{-1,1\}$ be a PTF of degree at most k and let $g\colon \{-1,1\}^n \to \{-1,1\}$ be any function. Show that if $\widehat{g}(S) = \widehat{f}(S)$ for all $|S| \le k$, then $f = g$.

7.9 Let $f\colon \{-1,1\}^n \to \{-1,1\}$ be expressible as a PTF over the collection of monomials $\mathcal{F} \subseteq 2^{[n]}$, that is $f(x) = \mathrm{sgn}(p(x))$ for some polynomial $p(x) = \sum_{S\in\mathcal{F}} \widehat{p}(S)x^S$. Show that $|S| \ge \alpha^{-1}$ for $\alpha = max_{S\in\mathcal{F}}|\widehat{f}(S)|$.