8.1 Let $f = \mathrm{OR}_n \circ \mathrm{AND}_n$. Compute $\mathrm{DT}(f)$, $\mathrm{C}^0(f)$ and $\mathrm{C}^1(f)$.

8.2 Show that $\mathrm{RT}(f) \geq \mathrm{bs}(f)/3$. Deduce that $\mathrm{DT}(f)$ and $\mathrm{RT}(f)$ are polynomially related.

8.3 Show that for any function $f\colon \{0,1\}^n \to \{0,1\}$ with $\mathrm{DT}(f) < n$ we have $\mathsf{dist}(f, \mathrm{XOR}_n) = 2^{n-1}$.

8.4 Show that for any f it is true that $\mathrm{DT}(f) \leq \mathrm{C}^1(f)\mathrm{bs}(f)$. Deduce that $\mathrm{DT}(f) \leq \mathrm{bs}(f)^3$.

8.5 Construct a function $f\colon \{0,1\}^n \to \{0,1\}$ with $\mathrm{bs}(f) = \Theta(n)$ and $\mathrm{s}(f) = \Theta(\sqrt{n})$.

8.6 Suppose $f\colon \{0,1\}^n \to \{0,1\}$ is monotone. Show that $\mathrm{s}(f) = \mathrm{bs}(f) = \mathrm{C}(f)$.

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8.7 Show that for $f\colon \{0,1\}^n \to \{0,1\}$ and for any k we have $\mathrm{C}^1(f) \leq k$ iff f can be represented as k-DNF. Analogously, $\mathrm{C}^0(f) \leq k$ iff f can be represented as k-CNF.

8.8 Deduce from the previous problem that if f is computable by k-DNF and m-CNF, then $\mathrm{DT}(f) \leq km$.

8.9 Show that the fraction of functions $f\colon \{0,1\}^n \to \{0,1\}$ with $\mathrm{DT}(f) < n$ tends to 0 with $n \to \infty$.