9.1 Show that there is a polynomial of degree $O(\sqrt{n})$ approximating AND_n with error $1/3$ and with rational coefficients both numerators and denominators of which are of absolute value $2^{O(n^{1/2}\log(n))}$.

9.2 Show that AND_n can be approximated with error $1/n$ by a polynomial of degree $O(\sqrt{n}\log n)$.

9.3 Show that there is a polynomial of degree $O(\sqrt{n}\log n)$ approximating AND_n with error $1/n$ and with with rational coefficients both numerators and denominators of which are of absolute value $2^{O(n^{1/2}\log^2 n)}$.

A *decision list* $L(x)$ of length k over variables $x_1, \dots, x_n$ is a sequence $(l_1, b_1), \dots, (l_k, b_k), b_{k+1}$, where each l_i is a literal and $b_i \in \{-1, 1\}$. Given $x \in \{0,1\}^n$ the value of $L(x)$ is b_i if i is the smallest index such that l_i is true on x; if all l_i's are false, then $L(x) = b_{k+1}$.

9.4 Show that any decision list of length k is computable by a PTF of degree 1 and weight $O(2^k)$.

9.5 Show that for any parameter h any decision list of length k is computable by a PTF of degree h and weight $O(2^{k/h+h})$.

9.6 Show that any decision list of length k is computable by a PTF of degree $k^{1/2}$ and weight $2^{O(k^{1/2})}$.

9.7 Show that any decision list of length k is computable by a PTF of degree $k^{1/3}\mathsf{polylog}(k)$ and weight $2^{O(k^{1/3}\mathsf{polylog}(k))}$. (Hint: combine several previous results.)

Problems for homework **Due: March, 14, 2019**

9.8 For $x \in \{0,1\}^n$ let $\mathrm{NAE}_n(x) = 0$ iff $x_1 = \ldots = x_n$. Show that $\deg_{1/3}(\mathrm{NAE}_n) = \Theta(\sqrt{n})$.

9.9 For $k \in \mathbb{N}$ denote $\mathrm{INDEX}(i_1, \dots, i_{k-1}, A) = A[i_1, \dots, i_{k-1}]$, where $1 \le i_j \le n$ for all $j \in [k-1]$ and A is a $k-1$ dimensional array of bits. Show that simultaneous k-party communication complexity of INDEX is $\Omega(n)$.