

WORKSHOP ON BIRATIONAL GEOMETRY
MOSCOW, 29–31 OCTOBER 2018
ABSTRACTS

Ivan Arzhantsev (HSE and MSU)

Commutative algebraic monoid structures on affine spaces

We study commutative associative polynomial operations $\mathbb{A}^n \times \mathbb{A}^n \rightarrow \mathbb{A}^n$ with unit on the affine space \mathbb{A}^n over an algebraically closed field of characteristic zero. A classification of such operations is obtained up to dimension 3. Several series of operations are constructed in arbitrary dimension. Also we explore a connection between commutative algebraic monoids on affine spaces and additive actions on toric varieties. This is a joint work with Sergey Bragin and Yulia Zaitseva.

Vladimiro Benedetti (ENS)

Calabi–Yau zero loci inside Grassmannians

Zero loci of sections of bundles over homogeneous spaces can be successfully used to construct interesting varieties: an example is given by Mukai’s work on Fano threefolds; another is given by the two maximal families of hyper-Kähler manifolds due to Beauville–Donagi and Debarre–Voisin. Motivated by the latter, we study zero loci of sections of homogeneous vector bundles over classical and exceptional Grassmannians. We restrict our attention to small dimensional loci with trivial canonical bundle (of type Calabi–Yau), for which a classification is possible; as a consequence, it turns out that among them the only hyper-Kähler fourfolds are those already cited.

Then, if time will permit, we will introduce a generalisation of zero loci, namely orbital degeneracy loci, that can be used to construct more Calabi–Yau varieties.

Olivier Benoist (ENS)

A real period-index theorem

Lang has conjectured that a quadratic form in at least 5 variables over the function field of a real surface with no real points has a nontrivial zero. After explaining geometric motivations for this question, we will sketch a proof relying on a real variant of de Jong’s period-index theorem, and based on Hodge-theoretic arguments.

Olivier Benoist (ENS)

Density of sums of three squares

Hilbert has proven that a real polynomial in two variables that takes only nonnegative values is a sum of four squares of rational functions. Cassels, Ellison and Pfister have shown

that this result is optimal: there exist such polynomials that are not sums of three squares of rational functions. In this talk, we will explain why those polynomials that can be written as sums of three squares are dense in the set of those that are nonnegative. The proof relies on the study of real Noether–Lefschetz loci.

Alexei Golota (HSE)

On entire holomorphic maps tangent to foliations on threefolds

We consider complex projective threefolds endowed with a codimension one holomorphic foliation. Let us assume that there exists a holomorphic map from \mathbb{C}^2 to our threefold such that its image is Zariski dense and tangent to the foliation. Under these assumptions we want to study the implications for the birational geometry of the threefold. The main conjecture is that the threefold cannot be of general type. This statement can be seen as a particular instance of the Green–Griffiths–Lang conjecture as well as a generalization of a celebrated result of M. McQuillan from 1998. In my talk I will describe a new strategy towards the above conjecture, based on the study of positivity of the conormal bundle to the foliation.

Vladimir Lazić (Universität des Saarlandes)

On Generalised Abundance

I will discuss a surprising generalisation of nonvanishing and semiample conjectures from various contexts. This is joint work with Thomas Peternell.

Konstantin Loginov (HSE)

On non-rational fibers of del Pezzo fibrations

It is well known that a cubic del Pezzo surface can degenerate into a cone over an elliptic curve in a non-singular family. We investigate when a del Pezzo surface (of any degree) can degenerate into a non-rational surface in a “reasonably good” family. By this we mean a del Pezzo fibration over a curve in the sense of the Minimal Model Program. We will show that such degenerations depend on the singularities of the total space of the fibration, and that there is a correspondence between certain degenerations and del Pezzo fibrations with an action of a cyclic group.

Daniel Loughran (University of Manchester)

Cubic surfaces over finite fields

Serre has asked what are the possibilities for the number of rational points on cubic surfaces over finite fields. In this talk we give a complete solution to this problem, building on special cases treated by Swinnerton-Dyer. We also consider the more general “Inverse Galois problem” for cubic surfaces over finite fields. This is joint work with Barinder Banwait and Francesc Fité.

Vladimir Popov (Steklov Mathematical Institute of RAS)

Compressing finite subgroups of Cremona groups

So far, in the studies of finite subgroups of Cremona groups, they were all considered on an equal footing. However, in reality it is necessary to consider some of them as “not basic”, since they are obtained from others by a standard “base change” construction. This leads to the problem of finding the subgroups in the classification lists, which are obtained by such a nontrivial change, or, in another terminology, are nontrivially compressible. The talk is aimed to discuss this topic.

Luca Tasin (Universität Bonn)

On the minimal model program for the moduli space of curves

In this talk I will report on a recent work with G. Codogni and F. Viviani in which we investigate the first possible steps of the minimal model program for the moduli space of stable pointed curves M . In particular, we show that such steps have a modular interpretation and we relate them to the so Hassett–Keel program, which predicts that the log canonical models of M (with natural boundaries) have also a modular interpretation.