Boolean Logic (exercises)

1. Write down the propositional formula which encodes the following reasoning:

   > *If capital investments remain constant, government expenses will raise or unemploy-ment will occur. If government expenses will not raise, taxes will be reduced. If taxes are reduced and capital investments remain constant, unemployment will not occur. Therefore, government expenses will raise.*

   Is this formula a tautology?

2. Which of the following formulae are satisfiable?

   (a) $(p \to q) \to (q \to p)$

   (b) $\neg(p \to q) \wedge \neg(q \to p)$

   (c) $(q \to (p \wedge r)) \wedge \neg((p \vee r) \to q)$

   (d) $\neg p \vee ((p \to q) \to p)$

3. Which of the following formulae are tautologies?

   (a) $(p \to q) \vee (q \to p)$

   (b) $(p \to q) \to ((p \to (q \to r)) \to (p \to r))$

   (c) $(p \to q) \wedge \neg(q \to p)$

   (d) $((p \to q) \to p) \to p$

4. Construct a propositional formula A depending on variables p, q, r such that:

   (a) A is true if and only if exactly one of p, q, r is true;

   (b) A is true if and only if at least two of p, q, r are true;

   (c) A is true if and only if either all p, q, r are true or exactly one of p, q, r is true.

5. Construct a formula A such that:

   (a) $(r \to A) \equiv (r \to (p \vee q))$ and $(A \to r) \equiv ((\neg p \vee q) \to r)$

   (b) $((A \wedge q) \to \neg p) \to ((p \to \neg q) \to A)$ is a tautology

6. Transform the following formulae into DNF and CNF.

   (a) $((p \to q) \to r) \to p$

   (b) $p \wedge ((q \vee r) \to s)$

   (c) $((((p \to q) \to \neg p) \to \neg q) \to \neg r) \to r$

7. Transform the following formulae into CNF and use the resolution method to determine whether each of them is satisfiable. If yes, provide a satisfying assignment.

   (a) $(p \to (q \wedge r)) \wedge (q \to (r \vee \neg p)) \wedge (q \vee r) \wedge (q \to p) \wedge ((q \vee r) \to \neg p)$

   (b) $(p \to (q \vee r)) \wedge (r \to s) \wedge (q \to (r \vee s)) \wedge (r \to \neg p) \wedge \neg(s \wedge r) \wedge (r \vee p) \wedge (q \to (s \to r))$

8. Transform negations of the following formulae into CNF and use the resolution method to determine whether each of the original formulae is a tautology.

   (a) $(p \to (q \to r)) \to ((p \to \neg r) \to (p \to \neg q))$

   (b) $((p \to q) \to (r \to \neg p)) \to (\neg q \to r)$

9. Construct a formula in 3-CNF (more precisely, a series of formula of growing size w.r.t. a natural parameter n) for which the saturation process of the resolution method yields exponentially many clauses.