Turing Machines (exercises)

1. Construct a deterministic Turing machine which calculates $x + 1$ from a natural x, where x is given

   (a) in unary notation (as $\underbrace{1 \ldots 1}_{x \text{ times}}$);   (b) in binary notation.

2. Prove, that the following set of words belongs to P (is polynomially decidable), by constructing a deterministic Turing machine with polynomial runtime which decides whether a word belongs to the set or not:

   (a) $\{\underbrace{0 \ldots 0}_{k \text{ times}}\underbrace{1 \ldots 1}_{k \text{ times}} \mid k \geq 1\}$;   (b) $\{ww \mid w \in \{0,1\}^*\}$;   (c) $\{\underbrace{0 \ldots 0}_{k \text{ times}}\underbrace{1 \ldots 1}_{k \text{ times}}\underbrace{0 \ldots 0}_{k \text{ times}} \mid k \geq 1\}$.

3. Write down the sequence of configurations of the Turing machine from Exercise 2a, running on the following input:

   (a) 00;   (b) 000111;   (c) 00111.

4. Consider the non-deterministic Turing machine defined by the following table of transitions:

Input symbol

		0	1	B
	q_0	$(q_0, 1, \rightarrow)$	$(q_1, 0, \rightarrow)$	—
State	q_1	$(q_1, 0, \rightarrow)$ or $(q_0, 0, \leftarrow)$	$(q_1, 1, \rightarrow)$ or $(q_0, 1, \leftarrow)$	$(q_2, B, \rightarrow)$
	q_2	—	—	—

(B stands for the blank symbol.)

   (a) Draw its transitions as a graph.
   (b) Which configurations are reachable by this machine, when started on input word 011?

Graphs (exercises)

5. The *degree* of a vertex is the number of edges connected to it. Can there be a graph with the following degrees of vertices?

   (a) 9 vertices of degree 3, 11 vertices of degree 4, and 10 vertices of degree 5 (and no other vertices)?
   (b) 2 vertices of degree 3 and 3 vertices of degree 2?
   (c) one vertex of degree 1, two vertices of degree 2, one vertex of degree 3, two vertices of degree 5?

   If yes, how many edges should this graph have?

6. A *Euler path* in a graph is a path which visits each *edge* exactly once.

   (a) Can a graph with the following degrees of vertices have a Euler path: 3 vertices of degree 3 and one vertex of degree 5?
   (b) Will a graph with the following degrees of vertices *always* have a Euler path: 2 vertices of degree 1, 10 vertices of degree 4, and 7 vertices of degree 6?

7. A *Hamiltonian cycle* in a graph is a cycle which visits each *vertex* exactly once. Does the following graph have a Hamiltonian cycle?