Home Assignment # 2 (Theoretical Midterm)

Deadline: Wednesday, October 6, 2021, anywhere-on-Earth.

You may either bring your answers in written form to the lecture on Wednesday, October 6, or send them to `sk@mi-ras.ru` (scan or high quality photo is fine).

1. (a) Translate the *negation* of the following formula into CNF:
   $(p \to (q \to r)) \to ((p \to \neg r) \to (p \to \neg q))$.

   (b) Apply the Resolution Algorithm to determine whether this CNF is satisfiable. If yes, write down *all* its satisfying assignments.

2. Let A be a formula constructed from variables p, q, r using only the following logical operations: $\vee$, $\wedge$, and $\to$ (but not negation). Could a CNF for A include the clause $(\neg p \vee \neg q \vee \neg r)$? If yes, provide an example; if no, explain why.

3. Could there exist a graph with the following degrees of vertices: (a) 4, 3, 3, 1 ? (b) 4, 3, 3, 2, 2 ? (c) 5, 4, 4, 2, 2, 1 ? If yes, provide an example; if no, explain why. Loops and parallel edges are not allowed.

4. Construct a graph with 10 vertices such that every vertex has degree 3 and any two vertices are connected by a path of not more than 2 edges. Loops and parallel edges are not allowed.

5. A graph has two vertices of degree 1 and several vertices of degree 10. Prove that the vertices of degree 1 are connected by a path in this graph. (Hint: suppose the contrary.)

6. Construct a deterministic Turing machine with polynomial runtime which decides whether a word belongs to the following language: $\{ww^R \mid w \in \{0,1\}^*\}$. Here w^R means w written in the reverse order: for example, $(00101110)^R = 01110100$.

7. For a given undirected graph $G = (V, E)$, construct a Boolean formula φ_G with the following properties: (1) φ_G is polynomially computable from G; (2) φ_G is satisfiable if and only if vertices of G can be colored in 3 colors with the following condition: For any edge $(u, v) \in E$, if u and v have different colors, then there exists $w \in V$ such that $(u, w) \in E$, $(v, w) \in E$, and w has the third color (different from both u and v). (Notice that the coloring is not obliged to be "correct" in the traditional sense, that is, two vertices of the same color may be connected by an edge.)