Graphs (exercises)

1. The *degree* of a vertex is the number of edges connected to it. Can there be a graph with the following degrees of vertices?

   (a) 9 vertices of degree 3, 11 vertices of degree 4, and 10 vertices of degree 5 (and no other vertices) ?

   (b) 2 vertices of degree 3 and 3 vertices of degree 2 ?

   (c) one vertex of degree 1, two vertices of degree 2, one vertex of degree 3, two vertices of degree 5 ?

   If yes, how many edges should this graph have?

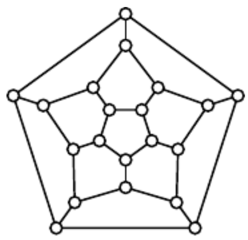
2. A *Euler path* in a (multi)graph is a path which visits each *edge* exactly once.

   (a) Can a multigraph with the following degrees of vertices have a Euler path: 3 vertices of degree 3 and one vertex of degree 5 ?

   (b) Will a multigraph with the following degrees of vertices *always* have a Euler path: 2 vertices of degree 1, 10 vertices of degree 4, and 7 vertices of degree 6 ?

3. Does the following decision problem belong to P: given a multigraph G, decide whether it has an Euler path?

4. A *Hamiltonian cycle* in a graph is a cycle which visits each *vertex* exactly once. Does the following graph have a Hamiltonian cycle?



5. *Reducibility.* Recall that, for two decision problems, $A \leq_m^P B$ if there exists a polynomial time computable function f such that $x \in A \iff f(x) \in B$, for any x.
   Consider the following decision problems:

   - INDSET: given a graph G and a number k, decide whether G contains an independent set of k vertices (that is, k vertices, none of which are connected);
   - CLIQUE: given a graph G and a number k, decide whether G contains a clique of k vertices (that is, k vertices, which are connected pairwise);
   - VERTEXCOVER: given a graph G and a number k, decide whether G has a vertex cover of k vertices (that is, a set U of k vertices such that for every edge at least one end of belongs to U).

   Show that: (a) INDSET $\leq_m^P$ CLIQUE and CLIQUE $\leq_m^P$ INDSET; (b) INDSET $\leq_m^P$ VERTEXCOVER.

6. Show that the 2-colorability problem for a graph (given a graph, determine whether it can be colored in two colors so that vertices connected by an edge have different colors) belongs to P, by reducing it to 2-SAT and using resolution method.