

**Home Assignment # 2 (Theoretical Midterm)****Deadline: Friday, October 14, 2022, anywhere-on-Earth.**Please submit by email to [slkuznetsov@hse.ru](mailto:slkuznetsov@hse.ru) (scan or high quality photo is fine).

1. Construct a Boolean formula with variables  $x, y, z, w$ , which is true if and only if at least 3 of these variables are true.
2. (a) Translate the *negation* of the following formula into CNF:  
 $(p \rightarrow (q \vee r)) \rightarrow ((r \rightarrow \neg p) \rightarrow (p \rightarrow q)).$   
(b) Apply the Resolution Algorithm to determine whether this CNF is satisfiable.
3. Let  $A$  be a formula constructed from variables  $p, q, r$  using only the following logical operations:  $\vee$  and  $\wedge$ . Could a DNF for  $A$  include the clause  $(\neg p \wedge \neg q \wedge \neg r)$ ? If yes, provide an example; if no, explain why.
4. Construct a graph with 10 vertices such that every vertex has degree 3 and any two vertices are connected by a path of not more than 2 edges. Loops and parallel edges are not allowed.
5. Seven schoolchildren were playing a chess tournament in one round (each plays one game with each). Before the lunch break Ivan played 6 games, Anatoly played 5 games, Alex and Dmitry played 3 games each, Simon and Ilya played 2 games each, and Eugene played only one game. Whom did Alex play with before the lunch break?
6. A graph (without loops and parallel edges) has 10 vertices and 20 edges. What is the maximal possible size of an independent set in such a graph? (Provide an example and prove that it is indeed the maximum.)
7. A graph has two vertices of degree 5 and several vertices of degree 24. Prove that the vertices of degree 5 are connected by a path in this graph. (Hint: suppose the contrary.)