Home Assignment # 2 (Theoretical Midterm)

Deadline: Sunday, October 15, 2023, anywhere-on-Earth.

Please submit by email to `slkuznetsov@hse.ru` (scan or high quality photo is fine).

1. Construct a Boolean formula with variables x, y, z, w, which is true if and only if at most 2 of these variables are true.

2. (a) Translate the *negation* of the following formula into CNF:
   $(p \to (\neg q \vee r)) \to ((r \to \neg p) \to (q \to \neg p))$.

   (b) Apply the Resolution Algorithm to determine whether this CNF is satisfiable.

3. Let A be a formula constructed from variables p, q, r using only the following logical operations: $\vee$ and $\wedge$. Could a CNF for $\neg A$ include the clause $(p \vee q \vee r)$? If yes, provide an example; if no, explain why.

4. Construct a graph with 10 vertices such that every vertex has degree 3 and any two vertices are connected by a path of not more than 2 edges. Loops and parallel edges are not allowed.

5. Seven schoolchildren were playing a chess tournament in one round (each plays one game with each). Before the lunch break Ivan played 6 games, Anatoly played 5 games, Alex and Dmitry played 3 games each, Simon and Ilya played 2 games each, and Eugene played only one game. Whom did Alex play with before the lunch break?

6. A graph (without loops and parallel edges) has 10 vertices and 20 edges. What is the maximal possible size of an independent set in such a graph? (Provide an example and prove that it is indeed the maximum.)

7. A graph has two vertices of degree 7 and several vertices of degree 100. Prove that the vertices of degree 7 are connected by a path in this graph. (Hint: suppose the contrary.)