

Home Assignment # 2 (Theoretical Midterm)**Deadline: Saturday, October 19, 2024, anywhere-on-Earth.**Please submit by email to slkuznetsov@hse.ru (scan or high quality photo is fine).

1. Construct a Boolean formula with variables x, y, z, w, t , which is true if and only if at most 3 of these variables are true.
2. (a) Translate the *negation* of the following formula into CNF:
 $(p \rightarrow (\neg q \vee r)) \rightarrow ((p \rightarrow q) \rightarrow (\neg r \rightarrow \neg p))$.
(b) Apply the Resolution Algorithm to determine whether this CNF is satisfiable.
3. Let A be a formula constructed from variables p, q, r using only the following logical operations: \vee and \wedge . Could a DNF for A include the clause $(\bar{p} \wedge \bar{q} \wedge \bar{r})$? If yes, provide an example; if no, explain why.
4. Construct a graph with 10 vertices such that every vertex has degree 3 and any two vertices are connected by a path of not more than 2 edges. Loops and parallel edges are not allowed.
5. Seven schoolchildren were playing a chess tournament in one round (each plays one game with each). Before the lunch break Ivan played 6 games, Anatoly played 5 games, Alex and Dmitry played 3 games each, Simon and Ilya played 2 games each, and Eugene played only one game. Whom did Alex play with before the lunch break?
6. A graph (without loops and parallel edges) has 10 vertices and 20 edges. What is the maximal possible size of an independent set in such a graph? (Provide an example and prove that it is indeed the maximum.)
7. A graph has two vertices of degree 21 and several vertices of degree 80. Prove that the vertices of degree 21 are connected by a path in this graph. (Hint: suppose the contrary.)