Home Assignment # 2 (Theoretical Midterm)

Deadline: Wednesday, October 22, 2025, anywhere-on-Earth.

Please submit by email to `slkuznetsov@hse.ru` (scan or high quality photo is fine).

1. Construct a Boolean formula with variables x, y, z, w, t, which is true if and only if no more than 3 of these variables are true.

2. (a) Translate the *negation* of the following formula into CNF:
   $(p \to (q \vee \neg r)) \to ((q \to t) \to ((r \vee t) \to (\neg t \to \neg p)))$.

   (b) Apply the Resolution Algorithm to determine whether this CNF is satisfiable. What does this mean for the original formula?

3. Let A be a formula constructed from variables p, q, r using only the following logical operations: $\vee$, $\wedge$, and $\to$. Could a CNF for A include the clause $(\bar{p} \vee \bar{q} \vee \bar{r})$? If yes, provide an example; if no, explain why.

4. Does there exist a coloured graph without loops and parallel edges with the following properties: it has 5 white and 6 black vertices; each edge connects vertices of different colour; all white vertices have the same degree; any two black vertices have different degree?

5. Seven schoolchildren were playing a chess tournament in one round (each plays one game with each). Before the lunch break Anatoly played 6 games, Ivan played 5 games, Simon and Ilya played 3 games each, Dmitry and Eugene played 2 games each, and Alex played only one game. Whom did Ilya play with before the lunch break?

6. Is the following formula of first-order logic satisfiable on a model with 10 elements? (Equality is interpeted in the standard way, as coincidence of elements; the interpretation of R is arbitrary.)

$$\forall x\,(\neg R(x,x)) \;\wedge\; \forall x \forall y\,(R(x,y) \to R(y,x)) \;\wedge\; \forall x \forall y\,(x = y \vee R(x,y) \vee \exists z\,(R(x,z) \wedge R(z,y)) \;\wedge$$
$$\forall x \exists y_1 \exists y_2 \exists y_3\,(R(x,y_1) \wedge R(x,y_2) \wedge R(x,y_3) \wedge \forall z\,(R(x,z) \to (z = y_1 \vee z = y_2 \vee z = y_3)))$$