## Turing Machines (exercises)

1. Construct a deterministic Turing machine which calculates $x + 1$ from a natural $x$, where $x$ is given

   (a) in unary notation (as $\underbrace{1\ldots1}_{x \text{ times}}$); (b) in binary notation.

2. Prove, that the following set of words belongs to $\mathsf{P}$ (is polynomially decidable), by constructing a deterministic Turing machine with polynomial runtime which decides whether a word belongs to the set or not:

   (a) $\{\underbrace{0\ldots0}_{k \text{ times}}\underbrace{1\ldots1}_{k \text{ times}} \mid k \geq 1\}$; (b) $\{ww \mid w \in \{0,1\}^*\}$; (c) $\{\underbrace{0\ldots0}_{k \text{ times}}\underbrace{1\ldots1}_{k \text{ times}}\underbrace{0\ldots0}_{k \text{ times}} \mid k \geq 1\}$.

3. Write down the sequence of configurations of the Turing machine from Exercise 2a, running on the following input:

   (a) 00; (b) 000111; (c) 00111.

4. Consider the non-deterministic Turing machine defined by the following table of transitions:

| State \ Input symbol | 0 | 1 | $B$ |
|---|---|---|---|
| $q_0$ | $(q_0, 1, \rightarrow)$ | $(q_1, 0, \rightarrow)$ | — |
| $q_1$ | $(q_1, 0, \rightarrow)$ or $(q_0, 0, \leftarrow)$ | $(q_1, 1, \rightarrow)$ or $(q_0, 1, \leftarrow)$ | $(q_2, B, \rightarrow)$ |
| $q_2$ | — | — | — |

   ($B$ stands for the blank symbol.)

   (a) Draw its transitions as a graph.
   (b) Which configurations are reachable by this machine, when started on input word 01 ?

## Graphs (exercises)

5. The *degree* of a vertex is the number of edges connected to it. Can there be a graph with the following degrees of vertices?

   (a) 9 vertices of degree 3, 11 vertices of degree 4, and 10 vertices of degree 5 (and no other vertices) ?
   (b) 2 vertices of degree 3 and 3 vertices of degree 2 ?
   (c) one vertex of degree 1, two vertices of degree 2, one vertex of degree 3, two vertices of degree 5 ?

   If yes, how many edges should this graph have?

6. A *Euler path* in a graph is a path which visits each *edge* exactly once.

   (a) Can a graph with the following degrees of vertices have a Euler path: 3 vertices of degree 3 and one vertex of degree 5 ?
   (b) Will a graph with the following degrees of vertices *always* have a Euler path: 2 vertices of degree 1, 10 vertices of degree 4, and 7 vertices of degree 6 ?

7. A *Hamiltonian cycle* in a graph is a cycle which visits each *vertex* exactly once. Does the following graph have a Hamiltonian cycle?