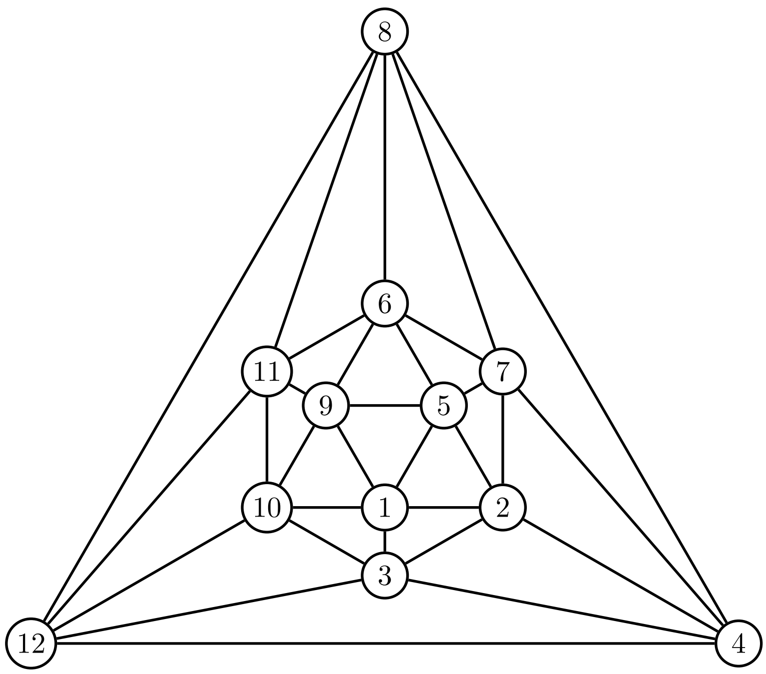
Final Exam

1. Construct a Boolean formula with three variables, p, q, and r, which is true if and only if at least two of these variables are assigned true. (This is called the *majority function:* the result is positive if and only if at least two of the three voters vote for it.)

2. Suppose a Boolean formula is constructed from variables (no constants) using only $\vee$ and $\wedge$. Could such a formula be a tautology? If yes, present an example. If no, explain why.

3. Let G be a graph with the set of vertices $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and let $\deg v_1 = \deg v_2 = 5$, $\deg v_3 = 4$, $\deg v_4 = 3$, $\deg v_5 = 2$. ($\deg v_i$ is the degree of v_i, that is, the number of edges connected to v_i. Parallel edges and loops are not allowed.) Draw the graph G and find $\deg v_6$.

4. Construct a Hamiltonian cycle on the following graph. (As the answer, please write down the sequence of vertices of the cycle.)



5. Find the minimal k for which the graph from Task 4 has a proper k-coloring. Provide a coloring of the graph's vertices in k colors, such that any two adjacent vertices have different colors, and explain why $(k-1)$ colors are not sufficient. (The coloring may be provided as a mapping, like "1 — red, 2 — blue, 3 — green, 4 — red, ...")

6. Suppose $\mathsf{P} \neq \mathsf{NP}$. Could there exist a polynomial-time algorithm which, given a Boolean formula φ with n variables, answers whether φ has *strictly more* than 2^{n-1} satisfying assignments?