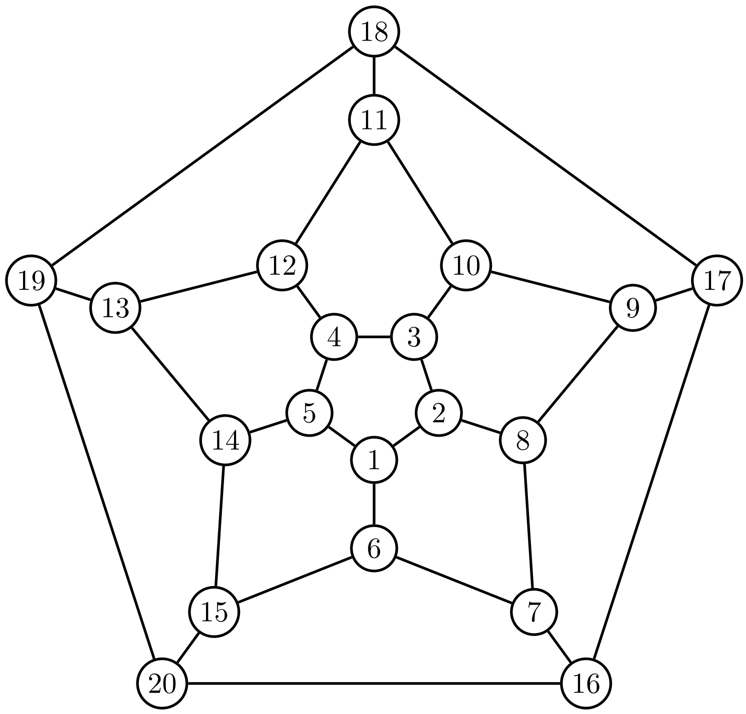
Final Exam

1. (1 point) Construct a Boolean formula with three variables, p, q, and r, which is true if and only if an odd number of these variables (one or three) are assigned true.

2. (1 point) Suppose a Boolean formula A is constructed from variables (no constants) using only $\rightarrow$ (implication). Could its negation $(\neg A)$ be a tautology? If yes, present an example. If no, explain why.

3. (1 point) A graph (without loops and parallel edges) has 10 vertices and 20 edges. What is the maximal possible size of an independent set in such a graph?

4. (2 points) Find the minimal k for which the graph below has a proper k-coloring. Provide a coloring of the graph's vertices in k colors, such that any two adjacent vertices have different colors, and explain why $(k-1)$ colors are not sufficient. (The coloring may be provided as a mapping, like "1 — red, 2 — blue, 3 — green, 4 — red, ...")



5. (2 points) For a given graph $G = (V, E)$, construct a Boolean formula φ_G of polynomial size, which is true if and only if there exists a coloring of *edges* of the graph in 3 colors, such that any two edges with a common end have different colors.

6. (2 points) Suppose $\mathsf{P} \neq \mathsf{NP}$. Could there exist a polynomial-time algorithm which, given a Boolean formula φ, answers whether the number of its satisfying assignments is greater or equal than 2? Explain your answer.

(One extra point is added just for the fact of participating in the exam. Participating means submitting something within the scope of the deadline.)