Final Exam

1. (1 point) Is the following Boolean formula a tautology?

$$\big((p \to (q \vee \neg r)) \wedge (q \to s) \wedge (\neg r \to (\neg s \to p)) \wedge (p \to \neg s)\big) \to \neg p$$

   If not, provide a falsifying assignment.

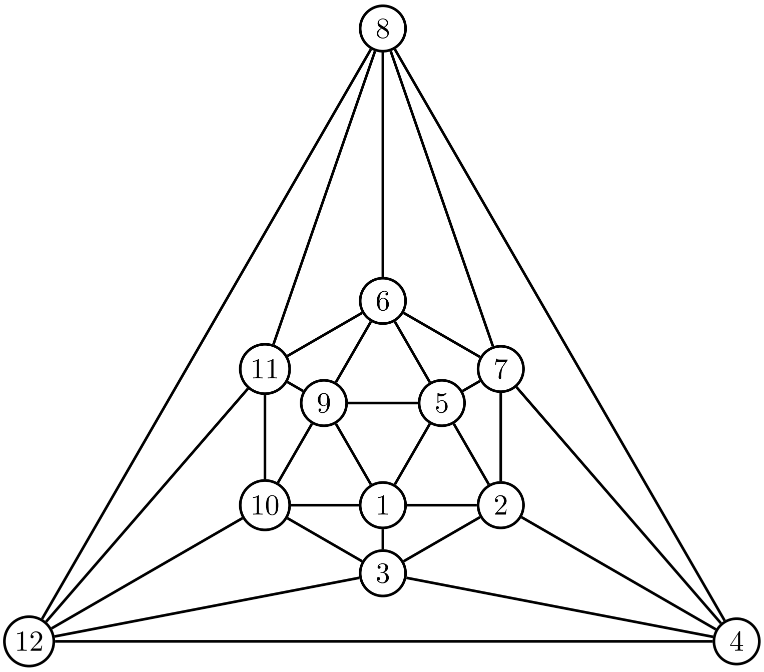
2. (1 point) Is the following first-order formula satisfiable?

$$\big(\forall x\, \exists y\, (R(x,y) \wedge P(y))\big) \wedge \big(\forall x\, \exists y\, (R(x,y) \wedge \neg P(y))\big)$$

   If yes, provide an interpretation of predicate symbols R and P on some set (domain) M, which makes the formula true.

3. (1 point) Does there exist a polynomial time algorithm which, being given a graph G, yields all Euler paths in G.

4. (2 points) Find the minimal k for which the graph below has a correct k-coloring. (A correct k-coloring is a coloring of vertices in k colors, such that ends of each edge have different colors.) Provide a correct k-coloring and explain why $(k-1)$ colors are not sufficient. (The coloring may be provided as a mapping, like "1 — red, 2 — blue, 3 — green, 4 — red, ...")



5. (2 points) The INTPROG problem (so-called "integer programming") is formulated as follows. Given a matrix $(a_{i,j})_{1 \le i \le n, 1 \le j \le m+1}$ of integers ($a_{i,j} \in \mathbb{Z}$), answer whether the system of inequations

$$\begin{cases} a_{1,1}x_1 + \ldots + a_{1,m}x_m + a_{1,m+1} \ge 0 \\ a_{2,1}x_1 + \ldots + a_{2,m}x_m + a_{2,m+1} \ge 0 \\ \ldots \\ a_{n,1}x_1 + \ldots + a_{n,m}x_m + a_{n,m+1} \ge 0 \end{cases}$$

   has an integer solution $(x_1, \ldots, x_m)$ (i.e., such $x_1, \ldots, x_m \in \mathbb{Z}$ that all inequations become true). Show that INTPROG is NP-hard by proving that 3-SAT $\le_m^P$ INTPROG.

6. (2 points) Suppose $\mathsf{P} \ne \mathsf{NP}$. Could there exist a polynomial-time algorithm which, being given a graph G, answers whether the number of correct 3-colorings of G is greater or equal than 3?

(One extra point is added just for the fact of participating in the exam. Participating means submitting something within the scope of the deadline.)