Final Exam

1. (1 point) Is the following Boolean formula a tautology?

$$\big((p \to (q \lor r)) \land (q \lor t) \land ((t \land p) \to w)\big) \to q$$

   If not, provide a falsifying assignment.

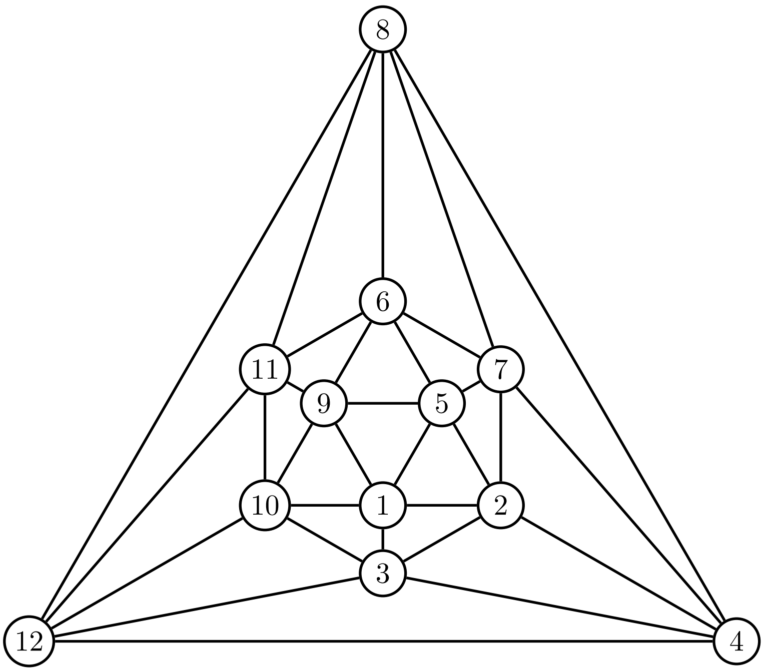
2. (1 point) Is the following first-order formula generally true?

$$\big(\exists x\, \forall y\, (R(x,y) \to P(y))\big) \lor \big(\exists x\, \forall y\, (R(x,y) \to \neg P(y))\big)$$

   If not, provide an interpretation of predicate symbols R and P on some set (domain) M, which makes the formula false.

3. (1 point) Does there exist a polynomial time algorithm which, being given a graph G, yields all correct colorings of G in 2 colors?

4. (2 points) Find the minimal k for which the graph below has a correct k-coloring. (A correct k-coloring is a coloring of vertices in k colors, such that ends of each edge have different colors.) Provide a correct k-coloring and explain why $(k-1)$ colors are not sufficient. (The coloring may be provided as a mapping, like "1 — red, 2 — blue, 3 — green, 4 — red, ...")



5. (2 points) Prove that the following problem is NP-complete: given a graph G with $3n$ vertices, determine whether G has a clique of n vertices.

6. (2 points) Suppose $\mathsf{P} \neq \mathsf{NP}$. Could there exist a polynomial-time algorithm which, being given a Boolean formula φ, answers whether the number of satisfying assignments for φ is less or equal than 2?

(One extra point is added just for the fact of participating in the exam. Participating means submitting something within the scope of the deadline.)