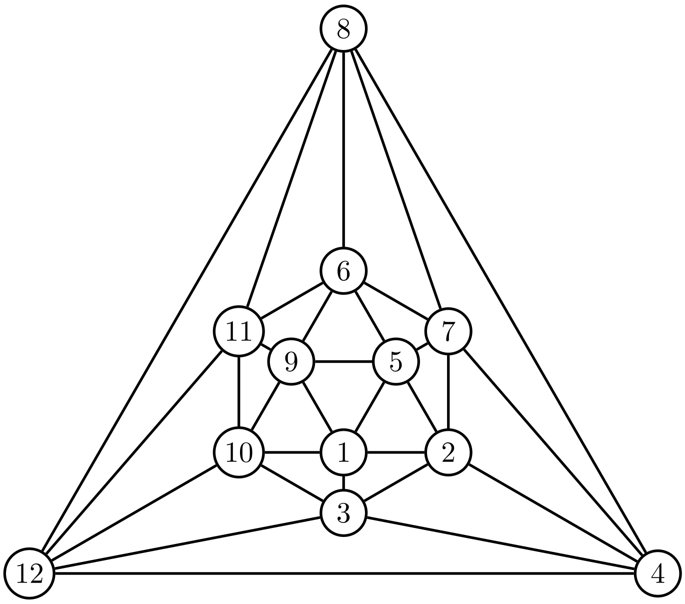
Final Exam

1. (1 point) Construct a Boolean tautology, using only one variable p, conjunction $\wedge$, and negation $\neg$.

2. (1 point) Let $A = (\neg(p \wedge q)) \to (q \wedge \neg r)$ and $B = (s \to p) \vee (s \to \neg r)$. Does there exist a formula C using only variables p and r (i.e., in the common language of A and B), such that both $A \to C$ and $C \to B$ are tautologies?

3. (1 point) Does there exist a polynomial-time algorithm that yields all Euler paths in a given undirected graph G?

4. (2 points) Find the minimal k for which the graph below has an vertex cover of k vertices. Provide an example of such a set (as a list of vertex numbers) and prove that for smaller values of k such an vertex cover does not exist.



5. (2 points) Is the following problem NP-complete? Given an undirected graph G, determine whether its vertices can be coloured into 4 colors, so that for each triangle (clique of 3 vertices) all its vertices are coloured differently.

6. (2 points) Suppose $\mathsf{P} \neq \mathsf{NP}$. Could there exist a polynomial-time algorithm which, given a directed graph G, determines whether G has *at least three* Hamiltonian cycles?
(*Comment:* informal argumentation like "this problem is obviously harder than HAMCYCLE, therefore not polynomially solvable" will not be accepted. There should be either a mathematical proof that the desired algorithm does not exist, or a polynomial-time algorithm presented.)

(One extra point is added just for the fact of participating in the exam. Participating means submitting something within the scope of the deadline.)