Home Assignment # 2

Deadline: Wednesday, October 16, 2020.

You are encouraged to bring your answers in written form to the class on Wednesday, October 14. If you're unable to attend this class or have not finished your assignment by October 14, please send a scan/photo to `sk@mi-ras.ru`

1. (a) Translate the *negation* of the following formula into CNF:
   $$(p \to (q \to r)) \to ((p \to \neg r) \to (p \to \neg q)).$$

   (b) Apply the Resolution Algorithm to determine whether this CNF is satisfiable. If yes, write down *all* its satisfying assignments.

2. Let A be a formula constructed from variables p, q, r using only the following logical operations: $\vee$, $\wedge$, and $\to$ (but not negation). Could a CNF for A include the clause $(\neg p \vee \neg q \vee \neg r)$? (If yes, provide an example; if no, explain why.)

3. Could there exist a graph with the following degrees of vertices: (a) 4, 3, 3, 1? (b) 4, 3, 3, 2, 2? (c) 5, 4, 4, 2, 2, 1? (If yes, provide an example; if no, explain why.)

4. Construct a graph with 10 vertices such that every vertex has degree 3 and any two vertices are connected by a path of not more than 2 edges.

5. A graph has two vertices of degree 1 and several vertices of degree 10. Prove that the vertices of degree 1 are connected by a path in this graph. (Hint: suppose the contrary.)

6. Construct a deterministic Turing machine with polynomial runtime which decides whether a word belongs to the following language: $\{ww^R \mid w \in \{0,1\}^*\}$. Here w^R means w written in the reverse order: for example, $(00101110)^R = 01110100$.