

## Home Assignment #1: Propositional Logic

1. Translate the following propositional formulae into Conjunctive Normal Form:
    - a.  $((((p \rightarrow q) \rightarrow \neg p) \rightarrow \neg q) \rightarrow \neg r) \rightarrow r$ ;
    - b.  $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow \neg r) \rightarrow (p \rightarrow \neg q))$ .
  2. Construct a formula  $A$  such that the formula  $(A \wedge q) \rightarrow \neg p \rightarrow ((p \rightarrow \neg q) \rightarrow A)$  is a tautology.
  3. Construct a formula with variables  $p, q, r$  that is true if and only if at least two of the variables are true.
  4. Construct a formula  $A$  such that  $(r \rightarrow A) \equiv (r \rightarrow (p \wedge q))$  and  $(A \rightarrow r) \equiv (\neg(p \vee q) \rightarrow r)$ .
  5. Use the resolution method to (dis)prove satisfiability of the formulae (in CNF) from Task 1.
  6. The formula  $A$  includes only variables  $p_1, \dots, p_k$  and the following logical operations:  $\vee, \wedge$ , and  $\rightarrow$  (but not the negation,  $\neg$ ). Prove that any CNF for  $A$  doesn't include the clause  $(\neg p_1 \vee \dots \vee \neg p_k)$ .
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### Deadline: Thursday, September 29.

Please bring your answers in written form to the class on Wednesday, September 28 or Thursday, September 29. If you're not going to attend these classes, please send a scan/photo to [sk@mi.ras.ru](mailto:sk@mi.ras.ru)