Discrete Mathematics for Algorithm and Software Design

## Home Assignment #2

**1.** Construct a deterministic finite automata for the language over alphabet  $\{c, d\}$  defined by the regular expression  $c(c^*d^*)^*d$ .

**2.** Consider the language over alphabet  $\{a, c\}$  consisting of all words of the form  $a^n ca^n ca^n$ , where  $n \ge 1$ . Can this language be defined by a regular expression? by a context-free grammar?

**3.** Construct a context-free grammar for the language over alphabet  $\{a, b\}$  consisting of all words of the form  $a^k b^m a^n$ , where k, m, n are natural numbers (zero is also a natural number!) and m = k + n.

4. Transform the following Boolean formula into CNF and apply the resolution method to determine whether it is satisfiable:

$$(p \to (q \land r)) \land (q \to (r \lor \neg p)) \land (q \lor r) \land (q \to p) \land ((q \lor r) \to \neg p).$$

If it is, provide a satisfying assignment.

5. Transform the following Boolean formula into CNF and apply the resolution method to determine whether it is satisfiable:

$$(p \to (q \lor r)) \land (r \to s) \land (q \to (r \lor s)) \land (r \to \neg p) \land \neg (s \land r) \land (r \lor p) \land (q \to (s \to r)).$$

If it is, provide a satisfying assignment.

**6.** Construct a Boolean formula A, with variables p, q, r, such that  $(p \to A) \equiv (q \to (\neg p \lor r))$  and  $(A \to p) \equiv ((r \to q) \to p)$ .  $(F_1 \equiv F_2 \text{ means that, for any assignment of 0's and 1's to variables, <math>F_1$  and  $F_2$  are either both true, or both false.)

7. Let A be a formula constructed from variables p, q, r using only the following logical operations:  $\lor$ ,  $\land$ , and  $\rightarrow$  (but not negation). Could a CNF for A include the clause  $(\neg p \lor \neg q \lor \neg r)$ ? (If yes, provide an example; if no, explain why.)

## Deadline: Wednesday, October 17, 2018.

Please bring your answers in written form to the class on Wednesday, October 17. If you're unable to attend this class, please send a scan/photo to sk@mi-ras.ru