

**Home Assignment # 2****Deadline: Wednesday, October 16, 2019.**

Please bring your answers in written form to the class on Wednesday, October 16. If you're unable to attend this class, please send a scan/photo to [sk@mi-ras.ru](mailto:sk@mi-ras.ru)

- Translate the following formula into CNF:  $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow \neg r) \rightarrow (p \rightarrow \neg q))$ .
  - Apply the Resolution Algorithm to determine whether this CNF is satisfiable. If yes, write down *all* its satisfying assignments.
- Let  $A$  be a formula constructed from variables  $p, q, r$  using only the following logical operations:  $\vee, \wedge,$  and  $\rightarrow$  (but not negation). Could a CNF for  $A$  include the clause  $(\neg p \vee \neg q \vee \neg r)$ ? (If yes, provide an example; if no, explain why.)
- Could there exist a graph with the following degrees of vertices: (a) 4, 3, 3, 1? (b) 4, 3, 3, 2, 2? (c) 5, 4, 4, 2, 2, 1? (If yes, provide an example; if no, explain why.)
- Construct a graph with 10 vertices such that every vertex has degree 3 and any two vertices are connected by a path of not more than 2 edges.
- A graph has two vertices of degree 1 and several vertices of degree 10. Prove that the vertices of degree 1 are connected by a path in this graph. (Hint: suppose the contrary.)
- Construct a deterministic Turing machine with polynomial runtime which decides whether a word belongs to the following language:  $\{ww^R \mid w \in \{0,1\}^*\}$ . Here  $w^R$  means  $w$  written in the reverse order: for example,  $(00101110)^R = 01110100$ .

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