S.P.Novikov

New Discretization of Complex Analysis (2 files):

File I: The Introduction

Few years ago we developed jointly with I.Dynnikov new discretization of complex analysis (DCA) based on the two-dimensional manifolds with colored black/white triangulation, Especially for the Euclidean plane with equilateral triangle lattice.

We started to develop a DCA theory for the analogs of equilateral triangle lattice in Hyperbolic plane. Some specific very interesting ”dynamical phenomena” appear in this case solving most fundamental boundary problems. Mike Boyle from the University of Maryland helped to use here the methods of symbolic dynamics.

History.

We do not discuss here "geometric" discretizations of conformal mappings started in early XX century. Our goal is to discretize Cauchy-Riemann operator $\bar{\partial}$ as a linear difference operator.

It was done first time by Lelong-Ferrand in 1940. Her discretization is based on the square lattice in R^2. Discrete version L of $\bar{\partial}$ acts on the C-valued functions of vertices $L\psi(m,n) =$

$$= \psi(m,n) + i\psi(m+1,n) -$$

$$- \psi(m+1,n+1) - i\psi(m,n+1)$$

The equation $L\psi = 0$ defines d-holomorphic functions.

Many people developed this approach. However, let me point out on the weak points in it: First, The operator L has order two because two length scales are involved (the lengths of the side and diagonal). Second, there is no natural factorization similar to $\Delta = \bar{\partial}\partial$, in the square lattice.

Discrete holomorphic functions do not form algebra neither in the classical approach nor in our new approach. How to define discrete analog of polynomials and rational functions?

There is no canonical solution in the standard approach based on the quadrilateral lattice but There are Canonical Analogs of Polynomilqs and Rational Functions in our case of the Equilateral Triangle Lattice Our approach is based on the ideas borrowed from the Complele Integrable Systems

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