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ABSTRACTS

Algebras of the p-adic distributions

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The p-adic Colombeau-Egorov algebra of generalized functions is constructed. For generalized functions operations of multiplication, the Fourier-transform, convolution, pointvalues are defined. The operations of (fractional) partial differentiation and (fractional) partial integration are introduced by the Vladimirov's pseudo-differential operator. In the standard Colombeau theory, generalized functions are *not uniquely* determined by their pointvalues [3]. Unlike this fact, in the p-adic theory generalized functions are *uniquely* determined by their pointvalues.

We introduce a concept of *associated homogeneous* p-adic distributions and provide a mathematical description of all associated homogeneous distributions and their Fourier-transform (see [1],[2]).

We construct an *associative and commutative* algebra of *asymptotic distributions* generated by the linear span of p-adic associated homogeneous distributions. This algebra is also *an associative and commutative* convolution algebra. Any element of this algebra is represented as a *weak asymptotic series* whose coefficients are associated homogeneous distributions. This algebra can be embedded as a subalgebra into p-adic Colombeau-Egorov algebra.

These results can be used to study some problems of the p-adic quantum mechanics with a singular potentials.

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p–Adic wavelet transform and quantum physics

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Abstract

p–Adic wavelet transform is considered as a possible tool for the description of hierarchic quantum systems.

Rolling Tachyon on non-BPS brane and p-adic string

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Abstract

A rolling solution to equations of motion of the tachyon field living on an unstable D-brane obtained in [1] within the level truncated string field theory,

$$D_t^{-2}\Upsilon = \Psi^2,$$

$$\left(-q^2\frac{d^2}{dt^2}+1\right)D_t^{-2}\Psi = \Upsilon\Psi,$$

is discussed. Here $\Psi = \Psi(t)$ is the tachyonic field, $\Upsilon = \Upsilon(t)$ is an auxiliary field and $D_t^{-2} = e^{\frac{1}{4}\frac{d^2}{dt^2}}$. As in the case of p-adic string [2] one can rewrite this system of differential equations in the integral form

$$C\Upsilon = \Psi^2,$$

$$C_q\Psi = \Upsilon\Psi,$$

where

$$\mathcal{C}\psi(t) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-(t-t')^2}\psi(t')dt', \qquad \mathcal{C}_q\Psi \equiv (-q^2\frac{d^2}{dt^2}+1)\mathcal{C}\Psi.$$

The rolling solution starts from the perturbative vacuum and approaches to one of stable vacua in infinite time and it can been obtained as a perturbation of a solution to p-adic string field equation(p=3). The distinguished feature of this solution is that the pressure for this solution has its minimum at zero time equal to the energy and goes to minus energy at infinite time.

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What We Would Like to Do Using Ultrametric Diffusion Models

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Abstract

An overview is given of modern applications of the concept of rugged landscapes in the macromolecular dynamics and the origin of life. We demonstrate characteristic landscapes of clusters, macromolecules, biopolymers, and the evolutional system, and discuss whether the ultrametric diffusion models may describe a random walk on such landscapes. The problems, which are needed in the development of p–adic analysis, are of our main interest.

p–adic models of macromolecular dynamics

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Abstract

We discuss the ultrametric diffusion in the context of molecular dynamics constrained by complex energy landscapes. As an example, we consider a random walk on the degenerate hierarchical landscapes and show that the corresponding master equation coincides with the p-adic pseudo-differential equation of ultrametric diffusion. Using this approach, we describe anomalously slow relaxation inherent to macromolecular systems, as well as the kinetic data for CO rebinding to myoglobin.

Localised solutions in the scalar field model with logarithmic nonlinearity

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Abstract

It is shown [1,2] that within the frameworks of the p-adic theory of non-archimedian particles and strings the local model of the scalar field with logarithmic nonlinearity appears when p approaches 1. This model supports a wide class of localised extended (soliton and oscillon) solution in D-dimensional space; their dynamical properties and quasiclassical quantization will be presented.

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p–Adic and Adelic Quantum Mechanics

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Abstract

p-Adic mathematical physics emerged as a results of efforts to find a non-archimedean approach to the spacetime and string dynamics at the Planck scale. One of its main achievements is a successful formulation and development of p-adic [1] and adelic []2 quantum mechanics, which may be regarded as a first step towards a systematic construction of adelic M/string theory. These quantum mechanics have complex-valued wave functions of p-adic and adelic arguments, respectively. As a result of p-adic effects in application of adelic quantum mechanics to minisuperspace cosmological models one obtains spacetime discreteness with the Planck length as the fundamental one. A brief review of various aspects of p-adic and adelic quantum mechanics will be presented (for a similar review, see Ref. [3] and references within). In particular, path integrals for quadratic Lagrangians will be reviewed. A few mathematical and physical problems relevant for some further investigations in this field will be noted.

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Shilov boundary and generalized divisors of zero

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Abstract: Let L be a field complete for a non trivial ultrametric absolute value and let $(A, \| \,.\, \|)$ be a commutative normed L-algebra with unity whose spectral semi-norm is $\| \,.\, \|_{si}$. Let $Mult(A, \| \,.\, \|)$ be the set of continuous multiplicative semi-norms of A and let $\mathcal{S}$ be the Shilov boundary for $(A, \| \,.\, \|_{si})$. Then $\mathcal{S}$ is equal to the set of the $\psi \in Mult(A, \| \,.\, \|)$ such that for every neighborhood $\mathcal{U}$ of ψ in $Mult(A, \| \,.\, \|)$, there exists $\theta \in \mathcal{U}$ and $g \in A$ satisfying $\|g\|_{si} = \theta(g)$ and $\gamma(g) < \|g\|_{si}\ \forall \gamma \in \mathcal{S} \setminus U$. Suppose A is uniform, let $f \in A$ and let $\mathcal{Z}(f) = \{\phi \in Mult(A, \| \,.\, \|) \mid \phi(f) = 0\}$. Then f is a generalized divisor of zero if and only if there exists $\psi \in \mathcal{S}$ such that $\psi(f) = 0$. Suppose A is complete. If fA is not closed, then f is generalized divizor of zero; if f is a generalized divisor of zero and if the interior of $\mathcal{Z}(f)$ is empty then fA is not closed. Let A be an affinoid L-algebra: if the interior of $\mathcal{Z}(f)$ is not empty, then f is a divisor of zero; any generalized divisor of zero of A actually is a divisor of zero. Let A be a Krasner algebra $H(D)$ without non trivial idempotents: an element $f \in H(D)$ is a generalized divizor of zero if and only if $fH(D)$ is a closed ideal; moreover, $H(D)$ is a principal ideal ring if and only if it has no generalized divisors of zero but 0 (this new condition adds to the well known set of equivalent conditions found in 1969).

Noncommutative p-tachyon

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Abstract

We make the tachyon field of p-adic string theory noncommutative by replacing ordinary products with noncommutative products in the exact effective action. Solitonic lumps corresponding to D-branes is obtained for all values of the noncommutative parameter θ. This is in contrast to usual scalar field theories in which the noncommutative solitons do not persist below a critical value of θ.

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ON THE CAUCHY PROBLEM FOR DIFFERENTIAL EQUATIONS IN A BANACH SPACE OVER THE FIELD OF p-ADIC NUMBERS

Let A be a closed linear operator on a Banach space $\mathfrak{B}$ with norm $\|\cdot\|$ over a non-archimedean field K of characteristic zero. Denote by $E(A)$ the space of entire vectors of exponential type for the operator $A: E(A) = \varinjlim_{\alpha>0} E_\alpha(A)$, where

$$E_\alpha(A) = \big\{x \in \bigcap_{n\in\mathbb{N}_0=\{0,1,2,\dots\}} \mathcal{D}(A^n) \Big| \exists c = c(x) > 0 \quad \forall k \in \mathbb{N}_0 \quad \|A^k x\| \le c\alpha^k\big\}.$$

We consider the Cauchy problem

$$\begin{cases} y^{(m)}(z) &= Ay(z) \\ y^{(k)}(0) &= y_k, \quad k = 0, 1\dots, m-1. \end{cases} \tag{1}$$

By a solution of (1) we mean a vector-function of the form $y(z) = \sum_{n=0}^{\infty} c_n z^n$, $c_n \in \mathfrak{B}$, $z \in K$, with non-zero convergence radius $r = r(y) = (\overline{\lim}_{n\to\infty} \|c_n\|^{1/n})^{-1}$, such that $y(z) \in \mathcal{D}(A)$ in the open disk $\{z : |z|_K < r\}$ and satisfies (1) there ($|\cdot|_K$ is the valuation in K).

Theorem 1. *Problem* (1) *is uniquely solvable if and only if* $y_k \in E(A)$.

If the operator A is bounded, then problem (1) is uniquely solvable for arbitrary $y_k \in \mathfrak{B}$. As was also shown in [1], in the case where the space $\mathfrak{B}$ is taken over the field $\mathbb{C}$ of complex numbers, problem (1) is solvable if and only if

$$\forall n \in \mathbb{N}_0 \quad \exists \alpha > 0 \quad \exists c > 0 : \|A^n y_k\| \le c\alpha^n (n!)^m, \quad k = 0, 1, \dots m-1.$$

Theorem 1 makes it possible to obtain, in particular, an analog of the Cauchy-Kovalevskaya theorem for systems of partial differential equations over the non-archimedean field K (compare with [2]).

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THE CHENTSOV-AMARI UNIQUENESS THEOREM IN NONCOMMUTATIVE INFORMATION GEOMETRY

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Abstract

One of the major results in classical information geometry, initiated by Chentsov[1] and reformulated by Amari[2], is the uniqueness of the Fisher information metric among all those derivable by differentiations twice from invariant divergence functionals(contrast functionals of two distributions p, q in $D_f(p,q) = \int f(\frac{q}{p}) p d\mu, f(1) = 0, f''(1) > 0$): "The Fisher information is the only metric introduced by invariant divergences, and $\pm\alpha$-connections are the only connections which are dual to each other with respect to this metric, where α is given by $\alpha = 2\frac{f'''(1)}{f''(1)} + 3$." Explicitly,

$$\begin{array}{lcl} \mathbf{I}\ f''(1) g_{ij}^{Fisher}(\theta) & = & \partial_i \partial_j S_f(p(\theta), p(\theta'))\Big|_{\theta'=\theta} = -\partial_i \partial_{j'} S_f(p(\theta), p(\theta'))\Big|_{\theta'=\theta} \\ \mathbf{II}\quad f''(1)\Gamma_{ij,k}^{(\alpha)}(\theta) & = & -\partial_i \partial_j \partial_{k'} S_f(p(\theta), p(\theta'))\Big|_{\theta'=\theta}; f''(1)\Gamma^{(-\alpha)}\ \{ij, k'\} \leftrightarrow \{i'j'k\}, \end{array}$$

where $\partial_i \equiv \frac{\partial}{\partial\theta^i}, \partial_{i'} \equiv \frac{\partial}{\partial\theta^{i'}}$etc for the parameter sets $\{\theta^i\}$ and $\{\theta'^i\}$ ($p(x, \{\theta^i\})$ is abbreviated by $p(\theta)$) to define a given statistical model. The purpose of this paper is to extend the theorem to a noncommutative(quantum) framework.

We show that, if the underlying information manifolds are extended to finite quantum states (i.e. finite dimensional matrix manifolds), the above uniqueness theorem holds by three modifications such that

(a) the f-divergence $D_f(p,q)$ is replaced by the quasi-entropy$S_f(\omega_\sigma, \omega_\rho)$ introduced by Petz[3](a) with p, q being replaced by two invertible density matrices ρ, σ, and $S_F(\rho, \sigma) = \mathrm{Tr}\rho^{1/2} F(\Delta_{\sigma,\rho})\rho^{1/2}$[3](b), and the convexity by the operator convexity

(b) each ∂_i is interpreted to be the respective Fréchet partial derivative

(c) $g^{Fisher}(\theta)$ in **I** is replaced by the class of Wigner-Yanase-Dyson metrics parametrized by α[4] with restriction $|\alpha| \leq 3$ due to the operator-convexity of the quasi-entropy.

With these modifications, we prove that **I** and **II** are true if and only if the metric is identical to one of the $g^{WYD}(\theta)$'s.

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Energy Conservation for p-adic and SFT string equations

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Abstract

It is shown that spatially homogeneous configurations of the open string tachyon field, [1]-[3]

$$(-q^2\partial^2+1)e^{\frac{1}{4}\partial^2}\tilde{\phi}=\tilde{\phi}^3, \quad \text{where} \quad \tilde{\phi}=\tilde{\phi}(t), \quad \partial=\frac{d}{dt}$$

($q=0$ corresponds to a p-adic string with $p=3$) posses the conserved energy functional

$$E(t)=\frac{q^2}{2}(\partial\phi)^2-\frac{1}{2}\phi^2+\frac{1}{4}(\tilde{\phi})^4+\frac{1}{8}\int_0^1 d\rho(e^{-\frac{\rho}{8}\partial^2}\tilde{\phi}^3)\overleftrightarrow{\partial}(e^{\frac{\rho}{8}\partial^2}\partial\tilde{\phi}),$$

where $\phi\equiv e^{\frac{1}{8}\partial^2}\tilde{\phi}$ and $A\overleftrightarrow{\partial}B=A\partial B-B\partial A$.

Separated representation, $E(t)=E_\phi(t)+E_\psi(t)$, for the conserved energy of open-closed tachyon fields, $\phi=\phi(t)$ and $\psi=\psi(t)$, which satisfy to the following system of equations [4, 5]

$$(-\partial^2+1)e^{2k\partial^2}\tilde{\phi}-\tilde{\phi}^2+c_2\tilde{\psi}-2(\tilde{\phi}\tilde{\psi})=0, \qquad (-\partial^2+4)e^{2m\partial^2}\tilde{\psi}+c_2\tilde{\phi}-\tilde{\phi}^2=0,$$

with

$$E_\phi(t)=-\frac{1}{2}\phi^2+\frac{1}{2}(\partial\phi)^2+\frac{1}{3}\tilde{\phi}^3+k\int_0^1 d\rho(e^{(2-\rho)k\partial^2}\{1-\partial^2\}\tilde{\phi})\overleftrightarrow{\partial}(e^{k\rho\partial^2}\partial\tilde{\phi}),$$

$$E_\psi(t)=-2\psi^2+\frac{1}{2}(\partial\psi)^2+\tilde{\psi}\{(\partial^2+4)e^{2m\partial^2}\tilde{\psi}\}+m\int_0^1 d\rho(e^{(2-\rho)m\partial^2}\{4-\partial^2\}\tilde{\psi})\overleftrightarrow{\partial}(e^{m\rho\partial^2}\partial\tilde{\psi}),$$

is obtained (for this system $\phi\equiv e^{k\partial^2}\tilde{\phi}$, $\psi\equiv e^{m\partial^2}\tilde{\psi}$).

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Analysis based on Dirichlet space theory on some extensions of Q_p

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Structural study on stochastic processes is largely depend on the language for describing randomness. Since stochastic processes on non-Archimedean metric space can not be diffusions, every stochastic process can travel only on jump-by-jump basis in such an environment.

A lesson from study on stochastic processes on the Euclidean space says that structural grip on jump processes is more difficult than diffusions in general. This stems from the facts that one point can be instantly brought to another by jump, even though they are separated in a large distance and that one may observe a similar outcome after accumulation of very small jumps. In contrast, on non-Archimedean metric space, any massive accumulation of very small jumps never create effect with large magnitude. This structural gap between those two spaces is so significant that it sometimes may remove the difficulty of structural study on jumps of stochastic processes on non-Archimedean metric space.

Theory of Dirichlet space is a useful language for revealing the global property of stochastic process such as recurrence and transience, or equivalently absence and existence of Green function. It could tell us some microscopic behaviours of the stochastic process associated with the Dirichlet space. In fact, we can detect whether every point is hit by the particle, by examining polarity of one point.

In this small article, we will introduce a class of Dirichlet spaces on the field of p-adic numbers and explore function spaces defined as counterparts of the Sobolev space.

Similarly to the Dirichlet space theory that provides a class of bilinear forms including classical Dirichlet integral $\frac{1}{2}\int_{\mathbf{R}^n}(\nabla u, \nabla v)dx$, we can consider a family of bilinear forms which covers the bilinear form

$$\mathcal{E}(u,v) = \int_{\mathbf{Q}_p} D^{\alpha/2}u(x)D^{\alpha/2}v(x)\mu(dx)$$

on $\mathbf{Q}_p$, where the difference operator D^{γ} is given by

$$D^{\gamma}u(x) = \frac{p^{\gamma}-1}{1-p^{-\gamma-1}}\int_{\mathbf{Q}_p}\frac{u(x)-u(y)}{\|x-y\|_p{}^{\gamma+1}}\mu(dy),$$

u stands for locally constant function and μ stands for the Haar measure on $\mathbf{Q}_p$.

Based on this idea, we can study a class of Dirichlet space with spacial inhomogeneity and demonstrate comparison principle taken for granted in analysis which arises naturally from two comparable infinitesimal generators. Some properties of non-linear capacity relevant to the counterparts of Sobolev spaces will be in our perspectives. We will see that infinitesimal generator in some nice class assigns Dirichlet space which is isometric to a Sobolev space determined by another infinitesimal generator.

$P-$adic dynamic systems governed by functions of the form $\frac{ax^2}{bx+1}$

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It is a known fact, that analytical functions play an important role in the complex analysis. In $p-$adic analysis rational functions play a very similar role - dynamic systems with the governing functions of such form were already considered. Thus, it is only natural to investigate behaviour of dynamic systems governed by similar functions over the field of $p-$adic complex numbers C_p. One of such dynamical systems of a particular form $f(x) = \frac{ax^2}{bx+1}$ $(a \neq 0, b \neq 0)$ is studied in the work by M. Khamraev.

Set $r_n = \frac{\|a\|_p^{n-1}}{\|b\|_p^n}$ and $S_r(N) = \{t \in C_p : \|t - N\|_p = r\}$ for $N \in C_p$.

Observe, that this dynamic system has two fixed points: $H_1 = 0$ and $H_2 = \frac{1}{a-b}$. Also - the point $\omega = -\frac{1}{b}$ is obviously a pole.

Sets Ω, Ψ, Σ are the sets of all the points in C_p that fall into the pole at some iteration step, all the cyclic points with the cycle of length greater that 1, all the points that fall into the fixed point H_2 respectively.

For the dynamic system of the form $f(x) = \frac{ax^2}{bx+1}$ we get the following theorem.

Theorem. If $\|b\| \, \|a\|$ then for $f(x) = \frac{ax^2}{bx+1} : C_p \to C_p$ the following is true:

A) The point H_1 is the attracting fixed point and its attractor $A(H_1)$ is such that $C_p \backslash \cup_{n=0}^{\infty} S_{r_n}(H_1) \subset A(H_1)$.

B) The set $C_p \backslash A(H_1)$ is not empty and $(\Omega \cup \Psi \cup \Sigma) \subset C_p \backslash A(H_1)$.

C) The point H_2 is the repelling fixed point for the dynamic system $f(x) : C_p \to C_p$.

Applications of p-adic and ultrametric analysis in quantum theory, probability theory, image processing, dynamical systems, and cognitive modelling

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Abstract

We present a review on applications of p-adic numbers in various fields:

1). Quantum models with p-adic valued functions.
2). p-adic valued probabilities
3). Image processing based on the use of ultrametric spaces
4). Discrete dynamical systems
5). p-adic and ultrametric models of menatal space and cognitive processes

Interaction Measures on the Space of Distributions over the Field of p-Adic Numbers

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Abstract

We construct measures on the space $\mathcal{D}'(Q_p^n)$, $n \leq 4$, of Bruhat-Schwartz distributions over the field of p-adic numbers, corresponding to finite volume polynomial interactions in a p-adic analog of the Euclidean quantum field theory. In contrast to earlier results in this direction, our choice of the free measure is the Gaussian measure corresponding to an elliptic pseudo-differential operator over Q_p^n. Analogs of the Euclidean $P(\phi)$-theories with free and half-Dirichlet boundary conditions are considered.

p–Adic pseudodifferential operators and applications

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Abstract

Different subjects of theory of p–adic pseudodifferential operators are discussed:

1) Construction of the basis of p–adic wavelets [1], which are compactly supported locally constant functions. The basis of p–adic wavelets is a basis of eigenvectors of the Vladimirov operator of p–adic fractional differentiation.

Other examples of bases of eigenvectors of the Vladimirov operator were considered in [2], [3].

2) Construction of a new wide class of p–adic pseudodifferential operators diagonal in the basis of p–adic wavelets, but not diagonal in the Fourier basis [4].

3) Application of p–adic pseudodifferential operators to replica symmetry breaking in theory of spin glasses and to dynamics of disordered systems. We show that the Parisi matrix (in the simplest case) can be described as a p–adic pseudodifferential operator [5], [6].

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Some applications of non-Archimedean analysis to sociology and database analysis

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Let Z be some set, $T = \{0, 1, ...\}$, and $U \subset Z^T$, i.e. U be some set of maps $u : T \to Z$. Denote by u_t the restriction of u to $[0, t]$, $\widetilde{U} = \{u_t \mid u \in U, t \in T\}$. Further, let ϕ be some real-valued map defined on $\widetilde{U}$ and such that $0 < \phi(u_{t+1}) < \phi(u_t)$ for each u, t. For every $u, v \in U$, $u \neq v$, denote $\gamma(u, v) = \min_{t \in T}\{t \mid u(t) \neq v(t)\}$. Set

$$d(u,v) = \begin{cases} 0, & \text{if } u = v, \\ \max[\phi(u_{\gamma(u,v)}), \phi(v_{\gamma(u,v)})], & \text{otherwise.} \end{cases}$$

It is easy to show that d is an ultrametric (non-Archimedean metric) on U. Let us give some examples.

1. Let ψ be some real-valued function defined on T and such that $0 < \psi(t+1) < \psi(t)$. Set $\phi(u_t) = \psi(t)$.

2. Let Z be at most countable and ξ_t be a stochastic process with time T and the state space Z. Suppose $\mathbb{P}\{\xi_0 = z_0, ..., \xi_t = z_t\} > 0$ for every $t \in T$ and $z_0, ..., z_t \in Z$. Let $U = Z^T$. We can consider U as the set of all paths of the process ξ_t. Set $\phi(u_0) = 1$ for all u and $\phi(u_t) = \mathbb{P}\{\xi_0 = u(0), ..., \xi_{t-1} = u(t-1)\}$ for $t > 0$. Another possible construction is $\phi(u_t) = \mathbb{P}\{\xi_0 = u(0), ..., \xi_{t-1} = u(t-1), \xi_t = u(t)\}$ for all t.

Such ultrametrics and other ones arise in sociology when studying problems of social classification, typologization and stratification, social mobility and dynamics, and in analyze of databases with non-numerical data. The ultrametric depends on ordering of variables in the sociological model or, respectively, ordering of the fields in the database. It is turned out, however, that under some conditions all ultrametrics are equivalent "almost surely" and "in the mean" with respect to some probability measure. We give various constructions of these measures. As a result, we can represent a given "usual" metric via some ultrametrics and discover "hidden" hierarchical structures in a social space.

Further, we discuss some problems of data analysis in ultrametric spaces. These problems are different from ones in traditional metric spaces. Some aspects of computer realization are discussed too.

On the linearization of non-archimedean holomorphic maps

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Abstract

Abstract: We present some new results on the local linearization of non-archimedean holomorphic maps. A holomorphic map is said to be locally linearizable at a fixed point if there is a convergent (formal) power series that conjugates the holomorphic map to its linear part in some neighborhood of the fixed point.

Cauchy-type integration over local fields and its applications

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Abstract

Cauchy-type integration over local fields is defined and investigated. Non-Archimedean analogs of holomorphic functions are introduced. Theorems about residues and holomorphisity are studied. Moreover, non-Archimedean analogs of Cauchy-Green, Martinelli, Leray, etc. theorems are proved. Their applications to operator theory and mathematical physics is discussed.

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Fuzzy Phase Space/Space-Time Geometry as Approach to Nonrelativistic Quantization

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Abstract

Novel quantization formalism which exploits phase space with fuzzy ordered set (Foset) geometric structure studied [1]. Foset A elements - fuzzy points (FP) a_i beside standard ordering - $a_j \leq a_k$, permits also fuzzy ordering (FO) (weak incomparability) - $a_j \wr a_k$ characterized by normalized weight $w_{jk} \geq 0$; if $a_j \leq \alpha_k$; $\quad w_{jk} = 0$ [2]. a_i FO to $x \in X^m$ ordered metricized subset elements described by distribution $w_i(x) \geq 0$, which corresponds to a_i X coordinate principal uncertainty σ_x. In nonrelativistic Foset phase space (FPS) the particle m state $|m\}$ corresponds to FP characterized by x, p distributions $w(x), w^P(p)$; $(w^Q(q)$ for arbitrary observable Q). For m free and potential $U(x)$ motion FPS weak structure induce correlations between w^{Q_i}, w^{Q_j} and restricts possible w^Q evolution. Together with $|m\}$ spectral decomposition in FPS it induces $|m\}$ representation equivalent to rigged Hilbert space $\mathcal{H}$; eventually for m Path Integral quantization ansatz reconstructed without use of Hilbert space axioms. It's argued that proposed formalism is physically more consistent than Quantum Logics on fuzzy sets [3]. Comparison of Fuzzy Geometry and proposed quantization with p-adic Quantum Mechanics calculus discussed [4].

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Symmetries of renormalization group flow in the hierarchical model

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Abstract

We describe some symmetry properties of renormalization group mapping in the hierarchical fermionic model.These properties lead to some non-trivial bifurcation phenomena . The commutative relation of renormalization group and Fourier transformations is generalized on the bosonic hierarchical model and even on the Euclidean models.

Metric structure of ultrametric spaces

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Abstract

In our work we have reconsidered the old problem of diffusion at the boundary of ultrametric tree from a "number theoretic" point of view. Namely, we use the modular functions (in particular, the Dedekind function) to construct the "continous" analog of the Cayley tree isometrically embedded in the Poincare upper half-plane. Later we work with this "continuous" Cayley tree as with a standard function of a complex vartiable. We have reproduced semi-analytically/semi-numerically the results of Ogielsky and Stein on dynamics on ultrametric spaces and have speculated on the new "geometrical" interpretation of replica $n \to 0$ limit.

Fuzzy cycles of p-adic dynamical systems

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Abstract

We consider the discrete dynamical systems in the field of p-adic numbers given by iterations of the monomial functions $x \mapsto x^n$. We find interesting algebraic and geometric properties of the dynamics. The number of fuzzy cycles are calculated. Some asymptotical results, $p \to \infty$ are also found.

The asymptotic number of periodic points of discrete p–adic dynamical systems

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Abstract

Let $\Phi_{r,f}(x)$ be the polynomial whose zeros are the r-periodic points of the polynomial $f(x) \in \mathbb{Z}[x]$. We will show that if $\Phi_{r,f}(x)$ is separable over $\mathbb{Q}$, then the average of numbers of linear factors of $\Phi_{r,f}(x)$ modulo $p\mathbb{F}_p[x]$ is equal to the number of orbits when the Galois group of $\Phi_{r,f}$ acts on the set of zeros of $\Phi_{r,f}(x)$. The Čebotarev Density Theorem and its application to determine the Galois group of a polynomial is discussed. If $f(x) = x^n$, where $n \geqslant 2$, then we are able to lift the result to the p-adic numbers, and explicit expressions are found of the average of numbers of linear factors of $\Phi_{r,f}(x)$ over the p-adic integers.

Poisson Algebra Homomorphisms and Poisson Brackets

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Abstract

It is shown that every almost unital almost linear mapping $h : \mathcal{A} \to \mathcal{B}$ of a unital Poisson Banach algebra $\mathcal{A}$ to a unital Poisson Banach algebra $\mathcal{B}$ is a Poisson algebra homomorphism when $h(xy) = h(x)h(y)$ holds for all $x, y \in \mathcal{A}$, and that every almost unital almost linear almost multiplicative mapping $h : \mathcal{A} \to \mathcal{B}$ is a Poisson algebra homomorphism when $h(2x) = h(2x)$, $h(3x) = 3h(x)$, or $h(qx) = qh(x)$ holds for all $x \in \mathcal{A}$. Here the numbers $2, 3, q$ depend on the functional equations given in the almost linear mappings.

We prove that every almost Poisson bracket $B : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ on Banach algebra $\mathcal{A}$ is a Poisson bracket when $B(2x, z) = B(x, 2z) = 2B(x, z)$, $B(3x, z) = B(x, 3z) = 3B(x, z)$, or $B(qx, z) = B(x, qz) = qB(x, z)$ for all $x, z \in \mathcal{A}$. Here the numbers $2, 3, q$ depend on the functional equations given in the almost Poisson brackets.

keywords: Poisson algebra homomorphism, Poisson bracket, Banach algebra, stability, linear functional equation

Fourier Transform of Distributions on Adeles

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Abstract

The talk concerns with distributions on group of adeles. The topics to be discussed are

- Adelic formulation of Riemann zeta functional equation: Tate formula, Volovich formula.
- Local finite part distribution and adelic finite part distribution, their Fourier transform.
- Distribution $\Delta_g(\psi) = \int_{\mathbb{A}^*} \psi(t)g(|t|)d^*t$, its regularisation and Fourier transform.
- Multi-index generalization of Tate distribution and associated Vladimirov operator.

Spline Approximation of Functions on Locally Compact Non-Archimedean Groups and Parisi Replica Matrices

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Let $p,\ q \in \mathbb{N} \setminus \{1\}$, $\mathbb{Z}_p$ and $\mathbb{Z}_q$ be rings of p-adic and q-adic integers respectively. Let f be a continuous function acting from $\mathbb{Z}_p \times \mathbb{Z}_q$ into the set of real numbers. Let $0 < \rho < 1$. We define the norm on $\mathbb{Z}_p$ and $\mathbb{Z}_q$ as following:

$$|x|_p = \rho^N \ (|y|_q = \rho^M) \ ,$$

where N (M) is the index of the first nonzero digit in the p-adic (q-adic) expansion of x (y). Then we can introduce the norm on $\mathbb{Z}_p \times \mathbb{Z}_q$ as

$$\|(x, y)\| := \max(|x|_p, |y|_q) \ .$$

We note that the norms $|\cdot|_p$, $|\cdot|_q$, $\|\cdot\|$ satisfy to the strong triangle inequality. For any $n \in \mathbb{N}$ let us consider the uniform partition of $\mathbb{Z}_p \times \mathbb{Z}_q$ with centers in $S = (\mathbb{Z}_p/p^n\mathbb{Z}_p) \times (\mathbb{Z}_q/q^n\mathbb{Z}_q)$. For this n we introduce the spline as

$$L_n(\overrightarrow{x}) = \sum_{\overrightarrow{i} \in S} \lambda(\overrightarrow{i}) \|\overrightarrow{x} - \overrightarrow{i}\| \ ,$$

where $\lambda(\overrightarrow{i})$ are unknown real numbers which can be derived from the system of linear equations

$$\sum_{\overrightarrow{i} \in S} \lambda(\overrightarrow{i}) \|\overrightarrow{j} - \overrightarrow{i}\| = f(\overrightarrow{j}) \ , \ \overrightarrow{j} \in S \ . \qquad (0.1)$$

If we enumerate S by an appropriate way then the matrix of system (0.1) will be Parisi replica matrix. Moreover it will be an invertible matrix. Conditions (0.1) mean a coincidence of L_n with f at points of S. Moreover L_n approximates f at $\mathbb{Z}_p \times \mathbb{Z}_q$ with respect to the uniform metrics.

Theorem. *For any $f \in C(\mathbb{Z}_p \times \mathbb{Z}_q \ ; \ \mathbb{R})$ the limit*

$$\lim_{n \to \infty} \max_{\overrightarrow{x} \in \mathbb{Z}_p \times \mathbb{Z}_q} |f(x) - L_n(x)| = 0$$

holds true.

ON PHASE TRANSITIONS FOR P-ADIC POTTS MODEL ON A CAYLEY TREE

FARRUH M. MUKHAMEDOV[1] UTKIR A. ROZIKOV[2]

It is known that one of the basic branches of mathematics which lies in the base of the theory of statistical mechanics is the theory of probability. Since in a non-archimedean case has been introduced the theory of probability [1] it naturally arises the study of the problems of statistical mechanics in the context of the p-adic theory of probability.

One of the central problem in the theory of Gibbs measures is to describe infinite-volume Gibbs measures corresponding to a given Hamiltonian. However, a complete analysis of the set of Gibbs measures for a specific Hamiltonian is often a difficult problem.

The Cayley tree Γ^k of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, from each vertex of which exactly $k+1$ edges issue. Let $\Gamma^k = (V, \Lambda, i)$, where V is the set of vertices of Γ^k, Λ is the set of edges of Γ^k and i is the incidence function associating each edge $l \in \Lambda$ with its endpoints $x, y \in V$. If $i(l) = \{x, y\}$, then x and y are called *neighboring vertices*, and we write $l = <x, y>$.

Let $\Psi = \{\sigma_1, \sigma_2, ..., \sigma_q\}$, where $\sigma_1, \sigma_2, ..., \sigma_q$ are elements of $\mathbb{Q}_p^{q-1}$ such that $\|\sigma_i\|_p = 1$, $i = \overline{1,q}$ and

$$\sigma_i \sigma_j = \begin{cases} 1, & \text{for } i = j, \\ 0, & \text{for } i \neq j \end{cases} \quad (i, j = \overline{1, q-1}), \ \sigma_q = \sum_{i=1}^{q-1} \sigma_i.$$

We consider p-adic Potts model where spin takes values in the set Ψ and is assigned to the vertices of the tree. Denote $\Omega = \Psi^V$, it is the configuration space on V. The Hamiltonian of p-adic Potts model has the form

$$H(\sigma) = -J \sum_{<x,y> \in L} \delta_{\sigma(x), \sigma(y)}, \quad n \in \mathbb{N},$$

here $\sigma = \{\sigma(x) : x \in V\} \in \Omega$, $|J|_p < p^{-1/(p-1)}$, $J \neq 0$ Here δ is the Kronecker symbol.

If the Gibbs measure for a given Hamiltonian non unique then we say that for this model there is a phase transition.

Theorem. *i) Let* $p = 2, q \in 2^2\mathbb{N}$ *and* $J_1 \neq 0$. *If* $q = 2^2 s$, $(s, 2) = 1$, $|J|_2 = \frac{1}{4}$ *or* $q = 2^m s, m \geq 3, (s, 2) = 1$, $|J|_2 \leq \frac{1}{4}$ *then there exists a phase transition for 2-adic Potts model on a Cayley tree of order 2.*
ii) Let $p \geq 3$, $q \in p\mathbb{N}$, *and* $0 < |J|_p \leq \frac{1}{p}$ *then there exists a phase transition for p-adic Potts model on a Cayley tree of order 2.*
iii) the p-adic Gibbs measure μ *corresponding to p-adic Potts model on the Cayley tree of order* k *is bounded if* $q \notin p\mathbb{N}$, *otherwise the one is not bounded.*

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p–Adic moments and Müntz Theorem

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Abstract

Let X be an infinite compact subset of the field $\mathbb{Q}_p$ of p-adic numbers. A subset S of $\{0, 1, 2, \ldots\}$ is called *a Müntz set for* X if every continuous function: $X \to \mathbb{Q}_p$ can uniformly be approximated by $\mathbb{Q}_p$-linear combinations of $x \mapsto x^n$ $(x \in X)$, where $n \in S$.
We determine the Müntz sets for $\{x \in \mathbb{Q}_p : |1-x| < 1\}$ and $\{x \in \mathbb{Q}_p : |x| = 1\}$. For the unit disk $\{x \in \mathbb{Q}_p : |x| \leq 1\}$ several sufficient conditions are derived for a set to be Müntz.

THE p–ADIC ENTROPY OF THE LOGISTIC MAP

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Abstract

The logistic map $x \mapsto kx(1-x)$ with rational k is considered on the projective line over the field of p-adic numbers. It is shown that its iterations demonstrate the chaotic behavior if and only if p divides the denominator of k. In this chaotic case the topological entropy is concentrated on the "filled Julia set" - it coinsides with the set of points whose coordinates remain integer p-adic numbers under iterations. The appropriate symbolic dynamics completely describes what happens on this set, and the corresponding binary entropy turns to be exactly 1.

The possible generalizations and the adelic interpretations of this result will be discussed.

Critical exponents in the bosonic hierarchical model

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Abstract

It is known that the hierarchical Dyson model may be considered as a discrete version of p-adic quantum field model. We describe non-Gaussian renormalization group invariant Hamiltonian in dimension 3 in terms of (4-d)-expansion and $(\alpha - 3/2)$-expansion, where d is the dimension of the hierarchical lattice and α is a renormalization group parameter. It is shown that both expansions (in low orders) have the same behaviour when p tends to infinity. We also compute the critical exponents in the hierarchical model in terms of both expansions.

Perturbed Dynamical Systems in $\mathfrak{p}$-adic Fields

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Abstract

Let k be a $\mathfrak{p}$-adic field, and let ßD be the class of all discrete dynamical systems, defined by polynomials of the kind $h(x) = x + g(x)$, where $g(x) \in k[x]$ is irreducible. By using Krasner's Lemma as a tool, we investigate the stability of this class with respect to perturbations of the kind $h_r(x) = h(x) + r(x)$, where $h(x) \in \sigma D$ and $r(x) \in k[x]$.

Keywords: $\mathfrak{p}$-adic fields, discrete dynamical systems, ramification, Krasner's Lemma

Some remarks on arithmetic quantum physics

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Abstract

This is a brief survey of results on arithmetic quantum physics. The subject has a long history going back to the works of Beltrametti, Ulam, Weyl, and others, but its recent history started with the suggestion of Volovich in 1987 that the geometry of spacetime is nonarchimedean at the Planck scale. Since that time many people have pushed this idea considerably. The talk will include brief discussions of Weyl systems, metaplectic groups and the work of Weil, harmonic oscillators, finite quantum systems and their limits, product formulae, and the relations between arithmetic subgroups and electromagnetic duality. Some problems are formulated which may be of interest for further progress.

Adelic formulas for 4-particles tree string and superstring amplitudes for one-class quadratic fields

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Abstract

Regularized adelic formulas for gamma- and beta-functions for arbitrary quasicharacters (either ramified or not) and for arbitrary field of algebraic numbers are concretized as applied to one-class quadratic fields (and to the field of rational numbers). Applications to 4-tachyon tree string amplitudes, to the (generalized) Veneziano amplitudes (open string) and to (perturbed) Virasoro amplitudes (closed string) as well as to massless 4-particle tree open and closed superstring amplitudes, to the Ramond-Neveu-Schwarz superstring amplitudes and to the charged heterotic superstring amplitudes are discussed. Certain relations between different string and superstring amplitudes are established. The problem of regularization of divergent infinite products of local gamma- and beta-functions is solved.

Remarks on p–adic Mathematical Physics. Summary and Some Future Directions

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Abstract

Summary of certain recent results in p-adic mathematical physics will be given. Some future directions related with p-adic quantum theory and string theory, non-commutative theory, complex systems and control theory, and quantum information are discussed.

On the Nonlinear Dynamical Equation in the p–adic String Theory

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Abstract

We study nonlinear pseudo-differential equations with infinite number of derivatives. This is a new class of equations which initially appeared in p-adic string theory. These equations are of much interest in mathematical physics and applications in particular in string theory and cosmology.

We perform [1] a systematical mathematical investigation of the properties of these equations. In the one dimensional case the canonical equation takes the form

$$p^{\frac{1}{2}\partial_t^2}\Phi = \Phi^p.$$

For this equation the main theorem of uniqueness in some algebra of tempered distributions is proved. Boundary problems for bounded solutions are studied; the existence of a space-homogenous solution for odd p is proved. For even p it is proved that there are no continuous solutions and it is pointed to the possibility of existence of discontinuous solutions. Multidimensional equation is also considered and its soliton and q-brane solutions are discussed.

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Trace formula on the p-adic upper half-plane and Prime geodesic theorem

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Abstract

The concept of the upper half-plane on a disconnected field was introduced by Gel'fand - Graev - Pyatetskii-Shapiro, and the half-plane is a homogeneous space with respect to the action of the special linear group over the field. We show an analogue of Selberg's Trace Formula on the upper half-plane on the p-adic field ($p \neq 2$), and describe the distribution of the "p-adic prime geodesics". For a torsion-free, discrete, and co-compact subgroup of $G = \mathrm{SL}(2, \mathbf{Q}_p)/\{\pm I\}$, the trace formula for finite graphs can be applied to study Ihara's zeta function. On the other hand, we consider hyperbolic, but not necessarily torsion-free subgroups of G, for which we represent Ihara's zeta function. We deal with Ihara's zeta function by the method of McKean where Selberg's zeta function was investigated via the heat kernel of Laplacian on a Riemann surface. In the work by McKean, the fundamental solution for Brownian motion on the surface played a key role, while for our p-adic case there exists no standard Brownian motion. We prepare a family of symmetric Markov processes on the p-adic upper half-plane, and among them employ two essentially distinct kinds of processes to derive the zeta function.

P–adic theory of stock market

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P-adic and adelic functional integral was applied to describe stock market. Analogy with strongly correlated condensed matter system (Hubbard and minority game models) was exploited. The wave function of market was described using supercoherent state of some version of minimally extended superlorents group which gave four states of traders (buy, sell, hold, shell). The measure of integration for the functional integral based on generalized supercoherent states for strongly interacting systems was calculated. This supercoherent state has two deformation parameters and describes quantum group.

As a result we have found that this functional integral for strongly nonlinear systems could be represented as the p-adic functional integral. The uncertainty in the choice of deformation parameters results in the fact that all the values of prime number p did exist in the integral and this gives us an adelic functional integral. As a starting point we have studied a quadratic form of the adelic lagrangian. The free adelic system was considered in the quasiclassical approximation. Numerical modelling of geodesics mapping (the solution of equations of motion for the free theory) from fractal space onto ordinary space was done for p=2,3,5 and for the basic types of curves (straight line, quadratic, cubic, exponential curves) and for their superposition as well. This modelling of geodesic trajectory revealed two type of p-adic waves in the space with fractal dimensions. One type of wave was similar to Eliott wave. An adelic wave was considered as geodesic one of the triangulated sphere. Computer program was worked out for the inverse mapping of the real time series of financial data for any fractal dimension equal to D. This method of analysis may be considered as the p-adic (or adelic) spectroscopy for fractals and multifractals time series.

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