RESEARCH SUMMARY

1. Topology of spaces of knots and plane curves. In [65], [9] I have constructed spectral sequences which calculate, in principle, all cohomology groups of spaces of knots and links in $\mathbf{R}^n$, $n \geq 3$, in particular (for $n = 3$) the knot invariants. The invariants derived from the efficient part of these calculations (so-called finite-order invariants or Vassiliev invariants) dominate all known polynomial invariants of links and Milnor's higher linking numbers. For $n > 3$ this spectral sequence gives immediately a calculation of all cohomology groups of the space of smooth links in $\mathbf{R}^n$. Later, using the concept of a *conical resolution* (cf. [36]) I have improved the general spectral sequence and calculated the homology type of spaces of knots realized by polynomial embeddings $\mathbf{R}^1 \to \mathbf{R}^n$ of degree ≤ 4, see [38]. In my 1991 lecture notes [66], see also [69], I have noticed that a very similar spectral sequence calculates homotopy invariants of links.

In [77] I have extended the main spectral sequence to one calculating the finite-order invariants of knots (and cohomology classes of higher dimensions of spaces of knots) in arbitrary 3-manifolds, including non-orientable and reducible ones, and presented examples of singular knot invariants, which cannot be integrated to usual knot invariants. (For orientable reducible manifolds, these invariants coincide with the ones studied previously by E. Kalfagianni.) The knot invariants arise as the elements of final terms $E_\infty^{-i,i}$ of this spectral sequence, however the spectral sequence itself (especially its higher differentials) is an invariant of the ambient 3-manifold. Also I showed that there are *integer-valued* finite-order knot invariants in all sufficiently complicated *non-orientable* 3-manifolds, distinguishing infinitely many different knots in them.

In [54]–[58] these invariants are applied to the study of *textile structures*, i.e. double-periodic links in $\mathbf{R}^3$ (which can be considered as links or knots in $T^2 \times \mathbf{R}^1$) or, more generally. This work was done at the Textile Engineering group of department of Art and Design, jointly with S.A.Grishanov, a textile professional who posed the technologically motivated problems. In [54]–[55] we introduce an infinite series of finite type knot invariants (of all positive degrees) in 3-manifolds of the form $M^2 \times \mathbf{R}^1$, $\pi_1(M^2) \neq 1$, and use them to separate the most classical textile structures. The invariants of degree k are defined by unordered collections of $k+1$ free homotopy classes of loops in $M^2 \times \mathbf{R}^1$; for $k = 1$ or 2 they essentially coincide with invariants defined previously by T.Fiedler. In [56] we construct explicit combinatorial formulas for these invariants, generalizing the similar Fiedler's formulas for degree 2 invariants. In [57], [58] we find (non-trivial) generalizations of these invariants and combinatorial formulas to the case of multi-component links. As an application, we find a topological obstruction for the reducibility of link and prove that many standard composite textile structures cannot be split.

In [69], [71], [10] I started the study of finite-order invariants of *ornaments*, i.e. collections of closed plane curves without triple intersections. Using a similar spectral sequence I found a large class of elementary invariants of such objects; the explicit calculation of this sequence leads to many natural problems in the modern homological combinatorics, cf. [78].

This theory applies perfectly to the classification of one-component plane immersed curves without triple intersections, developed by Arnold and extended by Polyak, Viro, Tabachnikov, Shumakovich, and Merkov, and also to more complicated theory of invariants of triple-points free plane curves with allowed singular points. This theory leads naturally to the study of *triangular* *diagrams* and *connected and two-connected hypergraphs* in the same way as the parallel knot theory leads to the calculus of chord diagrams and connected graphs. E.g., the simplest invariant of triple points free singular curves is of order 4 and is depicted by the triangular diagram  in the same way as the simplest knot invariant (of order 2) corresponds to the crossed 2-chord diagram $\bigoplus$, see [80].

In [78] I have calculated all groups of order ≤ 3 cohomology classes of spaces of knots in $\mathbf{R}^n$, $n \geq 3$. (In the case of odd n one of these classes, having order 3 and dimension $3n-8$ was found by my students D. Teiblum and V. Turchin in a computer experiment.) The very first of them is a $\mathbf{Z}_2$-valued $(n-2)$-dimensional cohomology class of order 1 in the space of compact knots in $\mathbf{R}^n$, defined as the linking number with the set of singular maps $S^1 \to \mathbf{R}^n$, gluing together some two *opposite* points of S^1. For even n similar class generates also the *integral* $(n-2)$-dimensional cohomology group (which is free cyclic). For $n = 3$ this class is nontrivial already in the component of unknots and proves that this component is not simply-connected.

In [47], [51] I have introduced a homological calculus for finding explicit combinatorial formulas for cohomology classes of spaces of knots in $\mathbf{R}^n$, generalizing the Polyak–Viro formulas for knot invariants in $\mathbf{R}^3$. In particular, in [47] I gave such an expression for the Turchin-Teiblum cocycle (reduced mod 2) and for all integral cohomology classes of orders ≤ 2. These expressions gave the first proof of the non-triviality of the Turchin-Teiblum cocycle in the space of long knots in $\mathbf{R}^3$, and also of an 3-dimensional cohomology class of degree 2 in the space of compact knots in $\mathbf{R}^3$. In [51] I have described a purely combinatorial algorithm of computing all finite-type knot invariants. This algorithm deals not with the planar pictures like the knot diagrams but with easily encodable objects like the chord diagrams; therefore it is ready for the efficient computer realization.

In [52], I have calculated all first degree cohomology classes of spaces of smooth generic immersions $\mathbf{R}^1 \to \mathbf{R}^n$, $n \geq 3$, with a standard behavior at infinity and prescribed transverse self-intersections; in particular all first degree invariants of such immersions for $n = 3$. Also, I have proved that any such cohomology class can be represented by a combinatorial formula generalizing the Gauß formula for linking numbers, containing only half-integer coefficients, and constructed the unique (and non-trivial) topological obstruction to the existence of similar combinatorial formulas with integer coefficients. As a corollary, I have proved that Polyak–Viro combinatorial formulas of certain knot invariants of degree 4 have necessarily non-integer coefficients. These homological calculations have led me to the formulation of an elementary criterion of planarity of Euler graphs, which was proved later by V.O. Manturov.

In [40] I have proved that any knot or link in $\mathbf{R}^3$ is isotopy equivalent to a holonomic one (i.e. given by the 2-jet extension of a differentiable function) and that the space of holonomic links in $\mathbf{R}^n$, $n > 3$, is homotopy equivalent to the space of all links. Later, J. Birman and N. Wrinkle have proved that any two holonomic knots, which are isotopy equivalent as usual knots, can be connected by a path in the space of holonomic knots.

2. Geometric combinatorics, theory of plane arrangements and matroids. In 1991, simultaneously with G. Ziegler and R. Živaljević, I have proved that the stable homotopy type of the complement of an arbitrary arrangement of affine planes depends only on the dimensions of (multiple) intersections of these planes; moreover, I gave an explicit expression for the homotopy type of the (Spanier–Whitehead dual to this complement) one-point compactification of the arrangement in the terms of these dimensions, see [95], [69]. The Goresky-MacPherson formula for the cohomology of the complement of the arrangement follows immediately from this expression.

In the joint work with V. Serganova, [33], we have constructed a matroid of rank three such that the space of its real realizations is non-empty but the dimension of this space is strictly less than that of the similar space of complex realizations. This example allowed us to prove some old problems in Singularity Theory, see § 6 below.

In the joint work with I.M. Gel'fand and A.V. Zelevinskii [26] we have calculated the homology groups of local systems on complements of a large class of complex hyperplane arrangements and gave an explicit geometric realization of their generators. (See also [13])

In [65], [90], [69] I started the study of the *complex of connected graphs* and calculated its homology groups, which are an important ingredient in the calculation of homology groups of spaces of knots. In [69], [71], [80] I started the study of complexes of *connected hypergraphs*, playing a similar role in the theory of generic plane curves, and in [78], [12] the study of complexes of *two-connected graphs,* providing an alternative approach to the homological study of knot spaces. These problems initiated a large series of works by specialists in Geometric Combinatorics (A. Björner, V. Welker, R. Simion, J. Shareshian, V. Turchin, a.o.).

In [36], [46] I have considered the *topological order complexes* of topologized partially ordered sets. In [42] I have used this notion to calculate the homology of some discriminant subvarieties in the spaces of polynomial functions $\mathbf{R}^2 \to \mathbf{R}^1$. In [44], also using this method, I have presented an algorithm of computing homology groups of spaces of nonsingular algebraic hypersurfaces in $\mathbf{CP}^n$ and calculated such groups for $d + n \leq 6$; e.g. the Poincaré polynomial of such a space with $d = 4, n = 2$ is equal to $(1 + t^3)(1 + t^5)(1 + t^6)$. This method was further developed by A.Gorinov to calculate the similar polynomial for $d = 5, n = 2$, and by O.Tommasi to calculate the cohomology groups of moduli spaces of (both usual and stable) complex curves of genre 4. In [50] I have applied this method to the study of spaces of Hermitian operators with simple spectra. For an overview of this method and these results, see also [109], [46], [82].

In [36] I have proved that the naturally topologized order complex of all proper subspaces in a vector space over $\mathbf{R}$, $\mathbf{C}$ or $\mathbf{H}$ is homeomorphic to a sphere.

3. Complexity theory. The *topological complexity* of a computational problem is the minimal number of branchings (operators IF) in algorithms solving it. Generalizing a work of S. Smale, I have found the best known and asymptotically sharp two-side estimate of the topological complexity of approximate solution of polynomial equations in one complex variable, see [28], [9].

In [39] I have proved that the topological complexity of approximate solution of real polynomial equations of degree d is equal to $d/2$ if d is even, and for odd $d \geq 3$ it also is positive (and is equal to 1 if $d = 3$ or 5).

In [31], [9] I have found best known estimates for complexity of approximate solution of similar problems on solving systems of polynomial equations in $\mathbf{C}^n$; it turned out that in all important cases the complexity is asymptotically proportional to the dimension of the space of systems.

In [41] I prove the following extension of the M. Rabin's theorem about recognizing the main orthant in $\mathbf{R}^n$: the depths of analytic decision trees recognizing the union of any r orthants in

$\mathbf{R}^n$ are estimated from below by the number $n - \mathrm{ord}_2 r$ (where $\mathrm{ord}_2 r$ is the number of twos in the primary decomposition of r), in particular by n if r is odd. As a corollary I prove that for any $d = 2^q$ and $l \in [1, d/2]$ the minimal depth of any analytic decision tree recognizing the set of real polynomials of degree d having at least $2l - 1$ real roots is no less than $d/2$.

A submanifold $M \subset \mathbf{R}^n$ is *r-neighborly*, if for any r points of M there is a hyperplane, supporting M and touching it at exactly these r points. In [43] I prove that the minimal dimension of the space $\mathbf{R}^n$, containing a stably r-supported k-dimensional submanifold, is asymptotically no less than $2kr - k$. This is the first general lower estimate in this problem, posed by M. Perles in the 1970-ies.

In [12], [34] I have proved an asymptotically sharp lower estimate of the minimal dimension of functional spaces whose elements interpolate any function $\mathbf{R}^n \to \mathbf{R}$ at any k points: if n is a power of 2 then this number lies in the interval $[k + (n-1)(k - d(k)), k(n+1)]$, where $d(k)$ is the number of ones in the binary representation of k. Similar estimates for functions on general manifolds are expressed in the terms of characteristic classes of configuration spaces.

4. Symplectic topology. In the works [21], [22], [1] I have constructed the universal chain complexes of singularities and multisingularities of wave fronts and caustics, which provide characteristic classes of Lagrangian and Legendrian manifolds, generalizing the Maslov index. They allow us to prove numerous restrictions on the coexistence and numbers of singular points on such manifolds.

Later, M. Kazarian has obtained a strong generalization and spectacular applications of these techniques to different problems of differential and symplectic topology.

In [45] I have proved that any compact group of symmetries of any real function singularity with finite Milnor number and zero 2-jet is discrete. This proves (modulo the results of Kazarian) that all rational Lagrange characteristic classes can be expressed in the terms of cohomology groups of our universal complex.

5. Theory of lacunas of hyperbolic PDE's. In 1983 I have proved that the sharpness (= the regularity of fundamental solutions) at points of wave fronts of almost all hyperbolic operators is equivalent to the local topological Petrovskiĭ condition (it was conjectured by Atiyah, Bott and Gårding, who introduced this condition in 1973) and demonstrated that for very degenerate fronts similar conjecture is false, see [99], [23], [24], [11]. I also have reduced the verification of the Petrovskiĭ condition to the computation of standard characteristics of singularities of functions, and found all local lacunas (domains of sharpness) close to "simple" (i.e. of types A_k, D_k, E_k) singularities of wave fronts, in particular close to all points of generic fronts in $\mathbf{R}^n$, $n \leq 7$, see [24], [7], [11]. In [35] I gave a simple geometrical characterization of local lacunas close to such singularities of fronts.

See also the last paragraph of the next § 6.

6. Theory of singularities. In my works on singularity theory several problems of the V.I. Arnold's 1976 and 1979 published lists of problems are solved, namely

- In our joint work with V. Serganova [33] we have proved that the number of real moduli of a singularity of a real function can be strictly less than the number of complex moduli of its complexification; this provides also a new example of a non-smooth $\mu = \mathrm{const}$ stratum of an isolated singularity.

- In [61] I have proved the stable irreducibility of classes of singularities and multisingularities in the parameter spaces of versal deformations of singularities of complex functions.
- In [61], [64] I have calculated stable cohomology rings of complements of discriminants and caustics of isolated singularities in $\mathbf{C}^n$. These rings are naturally isomorphic to the cohomology rings of the iterated loop spaces $\Omega^{2n}S^{2n+1}$ and $\Omega^{2n}\Sigma^{2n}U(n)/O(n)$ respectively. See also [27], [9], [10].

Also I have written a FORTRAN algorithm enumerating topologically distinct morsifications of singularities of real functions. Using this algorithm, I have found new local lacunas close to many singularities of wave fronts and proved the absence of such lacunas for some other singularities. See [11], [7].

7. Dynamical systems. In 1979 I have constructed the first example of a multidimensional pursuit problem, the dimension of whose attractor is twice greater than that of the target manifold of the problem; see pp. 219-220 of the expository article of Yu.S. Il'yashenko in Selecta Math. Sovetica, 1992, 11:3.

8. Generalized hypergeometric functions. In the joint works with I.M. Gel'fand and A.V. Zelevinskiĭ [25], [26] we have found the numbers of solutions of generalized hypergeometric systems on all sufficiently general strata of Grassmann manifolds, and proved that all of these solutions have integral representations.

9. Integral geometry and Picard–Lefschetz theory. Newton has proved that the area cut by a line from the domain bounded by a smooth plane curve cannot be an algebraic function of the cutting line. In 1987 I have proved a similar result for all convex domains in even-dimensional spaces, and found many obstructions to algebraicity in the odd-dimensional case (so that the Archimedes' example of an ellipsoid in $\mathbf{R}^{2n+1}$ becomes an exceptional phenomenon); these obstructions are formulated in terms of the local geometry of the complexification of the boundary of the domain. The proofs are based on methods of the (generalized) Picard–Lefschetz theory, see [30], [7], [11].

Studying the ramification of (the analytical continuations of) the volume function I found new generalized Picard–Lefschetz formulae, reducing the ramification of (both standard and intersection) homology groups of stratified singular varieties to the similar problem concerning the transversal slices of their strata, see [7], [37], [11], as well as similar reduction formulas for the ramification of homology (generally speaking, with coefficients in non-constant local systems) of complements of algebraic varieties depending on parameters, see [75].

10. Differential topology. In 1988 I have proved the "homological Smale–Hirsch principle" for the spaces of smooth functions $M \to \mathbf{R}^n$ without complicated singularities (i.e. without singularities defining subsets of codimension ≥ 2 in functional spaces); this theorem reduces the computation of the homology groups of such spaces to those of the corresponding spaces of admissible sections of the jet bundle $J^{\cdot}(M, \mathbf{R}^n) \to M$.

In 1985, working on a related problem (mentioned in the third paragraph of § 6) I have constructed a spectral sequence calculating the homology groups of such spaces, see [27], [61]; two simplest particular cases of this spectral sequence (corresponding to singularity classes defined by 0-jets of maps) coincide with the Adams–Eilenberg–Moore spectral sequence for loop spaces and the Anderson spectral sequence for spaces of maps of low-dimensional spaces into highly-connected ones.

Using similar techniques, I have proved in 1991 that the space of systems of k monic polynomials of degree d in $\mathbf{C}^1$ (or $\mathbf{R}^1$) without common roots is stably homotopy equivalent to the space of monic polynomials of degree $d \cdot k$ in $\mathbf{C}^1$ (respectively, $\mathbf{R}^1$) without roots of multiplicity k; this extends a theorem of F. Cohen, R. Cohen, B. Mann and J. Milgram establishing similar equivalence in the case $k = 2$. See [93], [70], [10].

In [42] for any d, k I have calculated the cohomology group of the space of homogeneous polynomials $\mathbf{R}^2 \to \mathbf{R}^1$ of degree d having no roots of multiplicity $\geq k$ in $\mathbf{RP}^1$. For $k = 2$ this problem gives us the first known example of the situation when finite-order invariants of something (in this case of smooth functions $S^1 \to \mathbf{R}^1$ without double zeros) do not constitute a complete system of invariants.

See also § 1 and [65].

11. Potential theory. Extending the famous theorems of Newton and Ivory about the potentials of spherical and elliptic layers, Arnold proved in 1982 that a hyperbolic layer in $\mathbf{R}^n$ does not attract a particle in the hyperbolicity domain. In [11], [76] I investigated the question for which d and n the potential of such a layer coincides with algebraic functions in other domains. I have proved that for $n = 2$ or $d = 2$ it is always so, and for $(n \geq 3)\&(d \geq 3)\&(n + d \geq 8)$ the potential of a generic hyperbolic surface of degree d in $\mathbf{R}^n$ is not algebraic in any domain other than the hyperbolicity domain.

For all values of n and d I have reduced this problem to the calculation of a certain subgroup of the monodromy group of some singularity of a complete intersection of codimension 2 in $\mathbf{C}^n$. Using this reduction, W. Ebeling has proved the similar non-algebraicity statement for remaining three cases $(d, n) = (3, 3), (3, 4)$ and $(4, 3)$.

12. Topology of Lie groups. In [36] I have constructed natural conical resolutions of determinant subvarieties in the spaces $GL(\mathbf{K}^n)$, $\mathbf{K} = \mathbf{R}, \mathbf{C}$ or $\mathbf{H}$. This construction (and the related natural filtration) gives an immediate realization of Miller's splitting of the homology of classical Lie groups into the homology of Grassmann manifolds, and allows to prove that the naturally topologized order complex of the set of all subspaces in $\mathbf{K}^n$ is homeomorphic to a sphere.

EDUCATION, PHILOSOPHY, PSYCHOLOGY, AND PSEUDO-SCIENCE

1. School education. Since 2005, I have read carefully about 150 Russian school textbooks in Mathematics for children of all years of education, and found in total above 9000 mistakes and incorrectnesses in them, see [138], [140], [142], [143] and also [136].

In [134], [135] I have analyzed the concept and scope of the PISA assessment of school education and found (besides many concrete mistakes in problems and answers) a scandalous failure in the opinion of related people concerning the role of Mathematics in raising a human being.

In [139] I analyze the scientific background of the antroposophic (aka Waldorf) pedagogical system, and show that it is insane.

2. IQ-tests. In [141] I investigate the logical problems in a book of IQ-tests, composed by the creator of the IQ-business, H.Eysenck. In particular, I show there that the author himself has solved correctly no more than 5 of 16 such problems presented in the book, while at least 11 problems are solved absolutely incorrectly. A comparable failure for geometrical problems contained in the same book is detected. On this basis, I make some investigation on what in fact (instead of the Intellect) these coefficients do measure. (See especially the complete electronic version of the article, referred to in the beginning of [141].)

3. Pseudo-science. In December 2009, I have actively participated in the scandal around the pseudo-scientific activity of V.Petrik, see [144], [145]. See also [139].

4. Philosophy. Marxist philosophical demagogy was a significant part of the communist ideology in Soviet Union. Former professors of it constitute a significant and influential social burden to the educational community, and strongly attempt to preserve their social value. One of these (successful) attempts was a new program of "candidate minimum" exams for post-graduate students, demanding that every person pretending to get PhD in Sciences or Mathematics should pass a very strong and detailed exam in Philosophy. In [53], [86], [137] I discuss the program and status of these exams (especially for Mathematicians) and the entire situation around them.

Concerning the philosophy of sciences, see also [144], [145].

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